Progress of FDC project

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- It is well known that precision theoretical description on high energy phenomonolgy must be achieved.
- Therefore, higher-order perturbative calculations in QFT for SM are required for signal and background.
- FDC project is aimed at automatic calculation on these calculation and already can do next-leading-order(NLO) calculation automatically.
- Based on FDC, there are already many hard works been achieved in last 8 years.
- Recent progress for FDC project will be introduced in this talk.

Brief Introduction to FDC package

Feynman Diagram Calculation(FDC).
This first version of FDC was presented at AIHENP93 workshp,1993.

FDC Homepage::
http://www.ihep.ac.cn/lunwen/wjx/public_html/index.html

FDC-LOOP
FDC-PWA
FDC-EMT

FDC-SM-and-Many-Extensions
FDC-NRQCD
FDC-MSSM

Written in REDUCE, RLISP,C++.
To generate Fortran

Event Generator

- The results are obtained analytically.
- Two ways to generate square of amplitude:
- Automatically phase space treatment
- To automatically construct the Lagrangian and deduce the Feynman rules for SM, MSSM
- First version of “FDC-LOOP” was completed by the end of 2007, used and improved since then.
- Many work on QCD correction are finished and published.
- First version of FDC-PWA was completed on 2001 and improved 2003, used by BES experimental group for partial-wave analysis

Automatically phase space treatment

It was presented at AIHENP96 and many improvements had been made

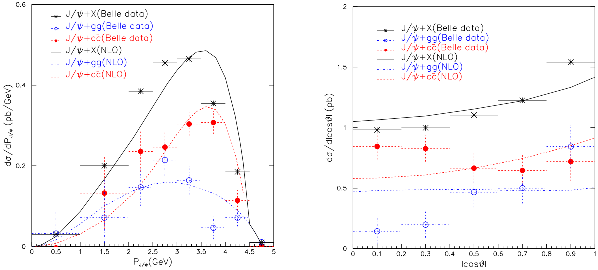
The program do analysis each Feynman diagram and look for:

t-channel peaks (calculate t_min, t_max)
s-channel peaks (calculate s_min, s_max)
sub-kinematics arrangement,
next sub-kinematics, ...

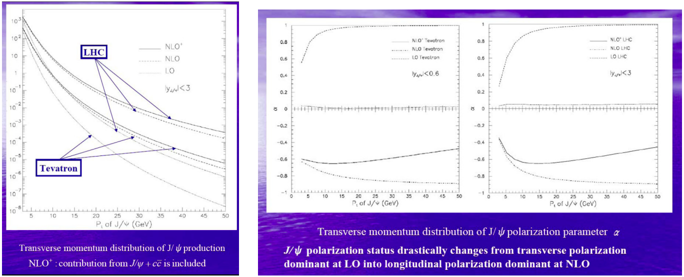
To generate Fortran source for these arrangement, and each sub-kinematics located in a sub-range. Sub-range divided by behave of Denominator of each diagram.

The calculations by using FDC-loop in last 8 years

- Our work concentrate on QCD correction to heavy quarkonium production and polarization in B-factory, z boson decay, Υ decay, HERA, Tevatron, LHC.
- It is found that that QCD corrections to these processes are very important.



Momentum distribution of J/ψ for $e^+e^- \to J/\psi + gg$ at QCD NLO. PRL102, (2009) B. Gong and J. X. Wang



P_t distribution of J/ψ polarization at QCD NLO. PRL100,232001 (2008), B. Gong and J. X. Wang

QCD Correction to prompt $J/\psi(^3S_1^1,\ ^1S_0^8,\ ^3S_1^8,\ ^3P_J^8)$ polarization

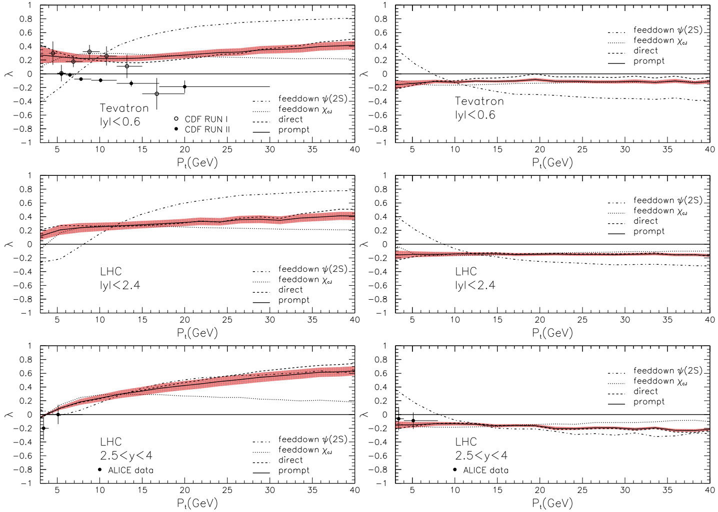
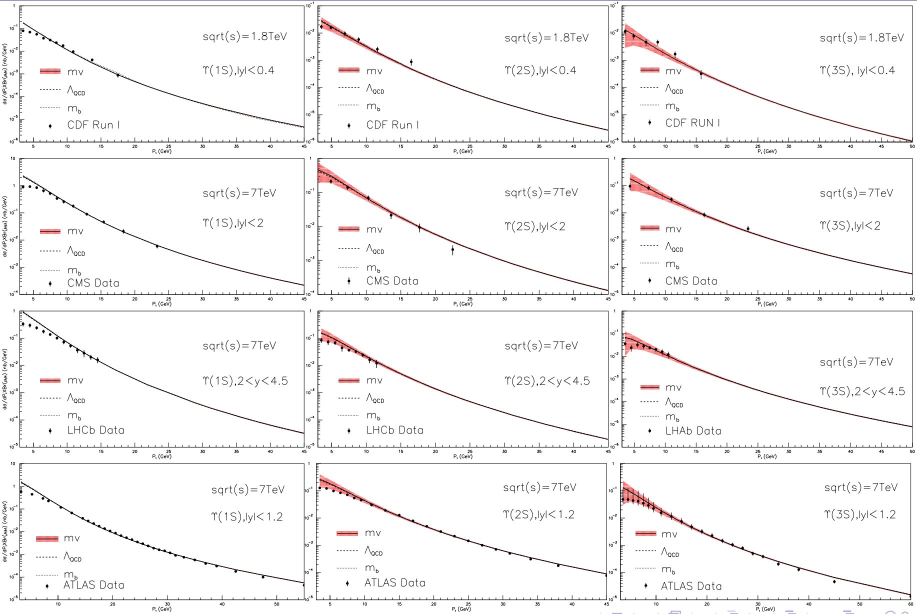


Figure: Polarization parameter λ of prompt J/ψ hadroproduction in helicity(left) and CS(right) frames.

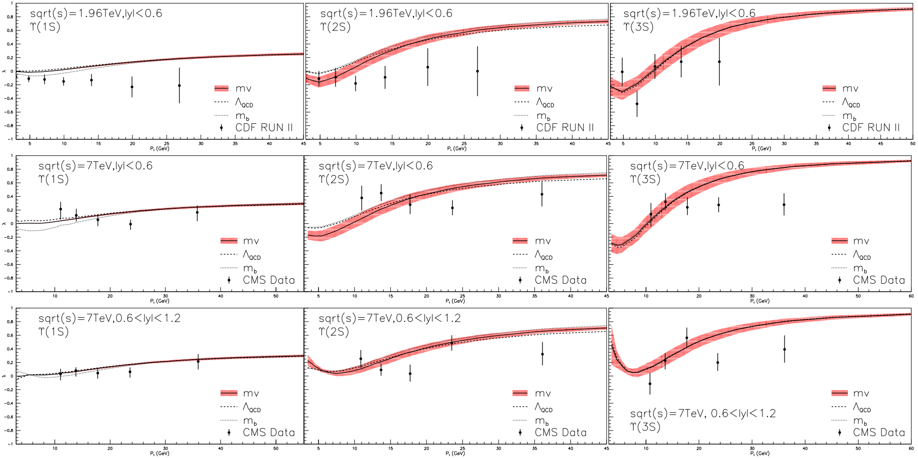
PRL110, 042002, 2013, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang

QCD Correction to $\Upsilon(1S, 2S, 3S)$ production



PRL 112, 032001, 2014, by Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang

QCD Correction to $\Upsilon(1S, 2S, 3S)$ polarization



PRL 112, 032001, 2014, by Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang

Figure: Polarization parameter λ of prompt $\Upsilon(1S, 2S, 3S)$ hadroproduction in helicity frame package includes

- 6 channels
- 76 sub-processes
- almost 2 millions lines Fortran codes in total.

n be run in paralleled mode with more than hundren thousands cpu with high efficiency.

STATES	LO sub-process	number of Feynman diagrams	NLO sub-process	number of Feynman diagrams
$^3S_1^{(1)}$	$g+g \to (Q\bar{Q})_n + g$	6	$g+g \to (Q\bar{Q})_n + g$(one-loop)	128
			$g+g \to (Q\bar{Q})_n + g + g$	60
			$g+g \to (Q\bar{Q})_n + Q + \bar{Q}$	42
			$g+g \to (Q\bar{Q})_n + q + \bar{q}$	6
			$g+q(\bar{q}) \to (Q\bar{Q})_n + g + q(\bar{q})$	6
$^1S_0^{(8)}$ (also $^3P_J^8$)	$g+g \to (Q\bar{Q})_n + g$	(12,16,12)	$g+g \to (Q\bar{Q})_n + g$(one-loop)	(369,644,390)
or	$g+q(\bar{q}) \to (Q\bar{Q})_n + q(\bar{q})$	(2,5,2)	$g+q(\bar{q}) \to (Q\bar{Q})_n + q(\bar{q})$(one-loop)	(61,156,65)
$^3S_1^{(8)}$	$q+\bar{q} \to (Q\bar{Q})_n + g$	(2,5,2)	$q+\bar{q} \to (Q\bar{Q})_n + g$(one-loop)	(61,156,65)
or			$g+g \to (Q\bar{Q})_n + g + g$	(98,123,98)
$^3P_J^1$			$g+g \to (Q\bar{Q})_n + q + \bar{q}$	(20,36,20)
			$g+q(\bar{q}) \to (Q\bar{Q})_n + g + q(\bar{q})$	(20,36,20)
			$q+\bar{q} \to (Q\bar{Q})_n + g + g$	(20,36,20)
			$q+\bar{q} \to (Q\bar{Q})_n + q + \bar{q}$	(4,14,4)
			$q+\bar{q} \to (Q\bar{Q})_n + q' + q'$	(2,7,2)
			$q+q \to (Q\bar{Q})_n + q + q$	(4,14,4)
			$q+q' \to (Q\bar{Q})_n + q + q'$	(2,7,2)

Table: The sub-processes for heavy quarkonium $c\bar{c}$ and $b\bar{b}$ prompt production at LO and NLO.

t.Phys.Commun. 18?:,2014, arXiv:1405.2143 by Lu-Ping Wan, Jian-Xiong Wang

Recent Progress in FDC project

- automatic counter terms generation, automatic calculation of renormalization constant (scheme dependent). . . .
- Geometric method for sector decomposition in multi-loop calculation

Geometric method for sector decomposition

- Sector Decompsotion is an old method and has been very actively developed. The most recently development is the Geometric method by T. kaneko and T. Ueda on 2010, Which beautifully and effectively translate the problem into a Geometric problem:
  "to construct and triangulate convex polyhedral cone"
- They (T. kaneko and T. Ueda) construct a computer program for their method by utilizing an algorithm from mathematican and they can finished the triple box(3-loop) by 53 hours CPU time.
- We develope a new algorithm and construct a program in Rlisp. We can finish the same work within 3-minuts CPU time on the same CPU.

Geometric method for sector decomposition

Sector Decomposition is a method used to separate divergences in loop integral.
With α presentation of a propagator

$$\frac{1}{D_l^{a_l}} = -i \int_0^\infty \mathrm{d}\alpha_l \exp(iD_l \alpha_l a_l),$$

An h-loop integral with N propagators can be expressed as

$$G = \int \frac{\mathrm{d}^d k_1 \mathrm{d}^d k_2 \cdots \mathrm{d}^d k_h}{D_1^{a_1} D_2^{a_2} \cdots D_N^{a_N}} = \int \mathrm{d}^d k \int \mathrm{d}^N \alpha \exp\left(i \sum_{l=1}^{N} D_l \alpha_l a_l \right)$$

After integration on loop momenta, it becomes

$$G = C \int_0^\infty \mathrm{d}^N \alpha \prod_l \alpha_l^{a_l - 1} U^{-d/2} \mathrm{e}^{-iF/U} \qquad (1)$$

where U and F are homogeneous polynomials of α_i with the homogeneity degrees h and $h+1$, and C is a constant.

Let $\eta = \sum \alpha_l$, insert $\delta(\eta - \sum \alpha_l)$ into the integral, and make the transformation $\alpha_l = \eta \alpha'_l$. After the integration over η, the integral becomes

$$G = C' \int_0^1 \mathrm{d}^N \alpha \delta \left(1 - \sum \alpha_l\right) \prod_l \alpha_l^{a_l - 1} \frac{U^{a-(h+1)d/2}}{F^{a-hd/2}} \tag{2}$$

with $a = \sum a_l$.

One can always reach Eq.(2) with usual loop integral techniques. And this is where sector decomposition starts. In this integral, only the integration over α_i is remained, and the interval is now limited to $[0, 1]$ due to the delta function.

And this is how sector decomposition works on it:

- separate the integration domain into N sectors $\Delta_{k,k=1,2,\ldots,N}$, where Δ_k is defined by $\alpha_i \leq \alpha_k, i \neq k$.
- do the transformation $\alpha'_i = \alpha_i/\alpha_k, i \neq k$ in Δ_k, and integrate over α_k with the delta function
- now, the integral in the integration domain Δ_k (labelled with G_k) becomes

$$G_k = C' \int_0^1 \mathrm{d}^{N-1} \alpha \prod_l \alpha_l^{a_l - 1} \frac{U_k^{a-(h+1)d/2}}{F_k^{a-hd/2}} \tag{3}$$

  where U_k and F_k are obtained by setting α_k to 1 in U and F.

- Usually these Δ_k are called primary sectors. But they are not sufficient since the divergences are still hidden inside.
- Further decomposition is needed.
- Here we introduce the geometric method [[**?**]], which can separate the divergence after one more decomposition (free from infinite recursion).
- For convenience, we rewrite G_k into

$$G_k = C' \int_0^1 \mathrm{d}^{N-1}\alpha \alpha^v U_k^\beta F_k^\gamma \tag{4}$$

with $v = \{a_1 - 1, a_2 - 1, \ldots, a_{N-1} - 1\}$, $\alpha^v = \prod_l \alpha_l^{a_l - 1}$, $\beta = a - (h+1)d/2$ and $\gamma = -(a - hd/2)$

The pure mathmatics problem for convex ployhedral cone

- perpendicularnormal vector, redundancy removal, hyperplane, original point 0,0,...)
- 1) To construct dual space: $\vec{y} \cdot \vec{v_i} \geq 0, \quad v_i \in S$ to find the domain of y in R^N. There must be solution(algorithm), Effaceny, ..
- 2) To triangulation of the dual space(convex ployhedral cone) into simplex (a convex ployhedral cone with N edges in N-dimension space)
- 3) We proved that to construct Dual space is equavilent to triangulation of the dual space into simplex
- 4) The vector space in step 1 in our physics applications are highly degenerated, therefore, optimized way can be developed.
- 5) The kernl algorithm is exactly the same, so the kernl program for 1) and 2) is the same one.

		Double box		Triple box	
Package	Strategy	No. of Sectors	Time(s)	No. of Sectors	Time(s)
FIESTA4	S	362	1.4	23783	831
	B	586	0.5	121195	127
	X	282	0.3	10259	44
	KU	266	13.7	6822	7472
	KU0	326	4.5	10556	6487
	KU2	266	61.9	-	-
SecDec3.0	X	320	1.4	11384	700
	G1	270	4.2	7871	574
T. Kaneko and T. Ueda		266	29.2	6568	123280
FDC-SD (Our result)		266	0.6	6568	116

Comparision with FIESTA4 [[**?**]], SecDec3.0 [[**?**]] and method proposers [[**?**]]. Strategies KU, KU0, KU2, G1 and G2 are all based on the geometric method, while G2 is different in the strategy of primary sectors.

decomposition only, without the integration of finite coefficients

Summary

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Thanks for your attention!