Automated calculation of matrix elements and physics motivated observables

Z. Was*,

*Institute of Nuclear Physics, Polish Academy of Sciences Krakow

- **(1)** Once computers arrived, for me it was year 1980, approach to phenomenology of theory/model based predictions could change a lot.
- **(2)** Numerous benefits became available. Drawbacks appeared as well. For example, methods of special functions expansions seem to be not as widespread as in the past.
- **(3)** I will concentrate on examples of my personal experience. I do not have any intensions to be systematic and balanced. Better picture will hopefully appear from other talks, e.g. examples of special functions expansions.
- **(4)** I will not focus on successes of the field. These are well known.
- **(5)** I will review traps which turned out to be rewarding to me once resolved; often in an unexpected way.

- Encouraged by Simizu-sensei conference, I choose to say what I always wanted, but never did.
- I thought the talk will be easy to prepare....
- In contrary, I found work frustrating, but rewarding.
- My plan is to show several simple examples of challenges resulting from complexity and how automated calculations were of help, but also a source of difficulties.
- Older examples originate from my work in Shimizu-san Minami Tateya group I visited in 1995.
- Each example in principle require substantial introduction, impossible to cover in one talk.
- My slides will show, outcome of my crippled attempts.

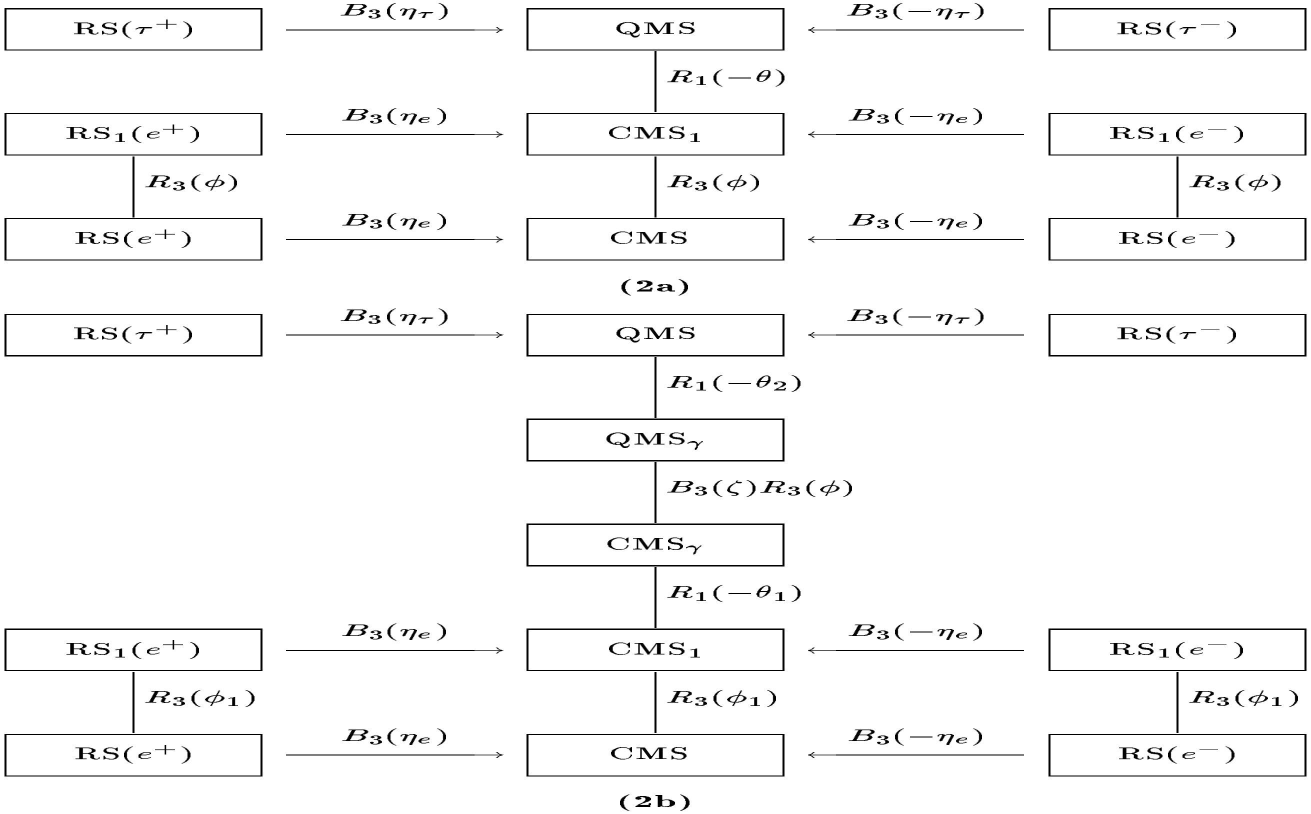
1983 Shoonship my first algebraic manipulation program

- At that time Poland was an isolated place, but with enormous in-flow of students to research. In reality a lot of contacts existed, but it was not to be seen by me.
- Access to computing was limited and in fact quite awkward: hopeless loss of time it seemed.
- One of my first project was to evaluate spin density matrix for the process $e^+e^- \to \tau^+\tau^-\gamma$ at Petra/PEP energies Monte Carlo Simulation of the Process e+ e- —> tau+ tau- Including Radiative O(alpha**3) QED Corrections, Mass and Spin S. Jadach, Z. Was (Jagiellonian U.). Mar 1984. Comput.Phys.Commun. 36 (1985) 191.
- Thhis work was performed under guidance of Prof. S. Jadach.
- Fantastic experience in looking at spin amplitudes as (reducible) representations of (Lorentz$\times$gauge) groups.
- It was great that we could spend all necessary time to understand details of what we were doing.
- In this particular case, how to represent moderatly complicated formulas of spin states into compact forms, exploiting geometrical properties of formulae.

1983 Shoonship my first algebraic manipulation program

To simplify and to understand amplitudes:

Figure 2



35

In the last formula the index $i = 1, 2, 3$ numbers the three components of $\vec{w}_1$ in the rest frame of the τ^+ lepton and the $k = 1, 2, 3$ numbers the axes in the rest system of the τ^- lepton. In both rest frames the third axis is the spin quantisation axis as in the definition of α_1 and α_2 and the first axis is defined to be perpendicular to the reaction plane i.e. along τ-vector. In (2.5) the absence of terms linear in w_k like $\sum_k R^0_{0k}\, w^k_2$ and $\sum_k R^0_{i0}\, w^i_1$ means that each $\tau^\pm$ separately is not polarized in the lowest order. There are, however, correlations between w_1 and w_2 which are controlled by the matrix R^0_{ik}. We extend the matrix R^0_{ik} to R^0_{ab} with $a, b = 0, 1, 2, 3$ obtaining

$$R^0_{ab} = \begin{bmatrix} 1+c^2+M^2s^2, & 0, & 0, & 0 \\ 0, & -(1-M^2)s^2, & 0, & 0 \\ 0, & 0, & (1+M^2)s^2, & 2Mcs \\ 0, & 0, & 2Mcs, & 1+c^2-M^2s^2 \end{bmatrix}. \tag{2.6}$$

In order to calculate the matrix R^0_{ab} as given by equation (2.6) directly from our spin amplitudes defined in Eq. (2.3), we have to translate the bispinor indices in the joint density matrix given by

$$\varrho^0_{\alpha_1\bar{\alpha}_1\alpha_2\bar{\alpha}_2} = \tfrac{1}{4} \sum_{\lambda_1\lambda_2} M^0_{\lambda_1\lambda_2\alpha_1\alpha_2} (M^0_{\lambda_1\lambda_2\bar{\alpha}_1\bar{\alpha}_2})^*$$
$$= \tfrac{1}{2} U^2 \left[|\alpha_+\bar{\alpha}_+| + \alpha_+\bar{\alpha}_+ c^2 + M^2 s^2 \alpha_-\bar{\alpha}_- - \frac{i}{2}(\alpha_+\bar{\alpha}_- - \alpha_-\bar{\alpha}_+) 2Mcs \right] \tag{2.7}$$

into vector indices a and b, see (2.6). The answer may be read off from Eq. (2.4) by substituting in the operator $\Lambda_\pm(p, w)$ as polarisation vectors the three space-like vectors $\hat{e}_1 = (0, 1, 0, 0)$, $\hat{e}_2 = (0, 0, 1, 0)$ and $\hat{e}_3 = (0, 0, 0, 1)$ and comparing the results with the bispinor quantities $u(p, \alpha)\bar{u}(p, \bar{\alpha})$ in the τ rest frame, $p = (M, 0, 0, 0)$, α being the spin projection onto $\vec{e}_3$. The result is

$$\begin{aligned} \Lambda_+(p, \hat{e}_1) - \Lambda_+(p, 0) &= \tilde{\Lambda}_+(p, e_1) = u(p,+)\bar{u}(p,-) + u(p,-)\bar{u}(p,+), \\ \Lambda_+(p, \hat{e}_2) - \Lambda_+(p, 0) &= \tilde{\Lambda}_+(p, e_2) = iu(p,-)\bar{u}(p,+) - iu(p,+)\bar{u}(p,-), \\ \Lambda_+(p, \hat{e}_3) - \Lambda_+(p, 0) &= \tilde{\Lambda}_+(p, e_3) = u(p,+)\bar{u}(p,+) - u(p,-)\bar{u}(p,-), \end{aligned} \tag{2.8}$$

and in addition

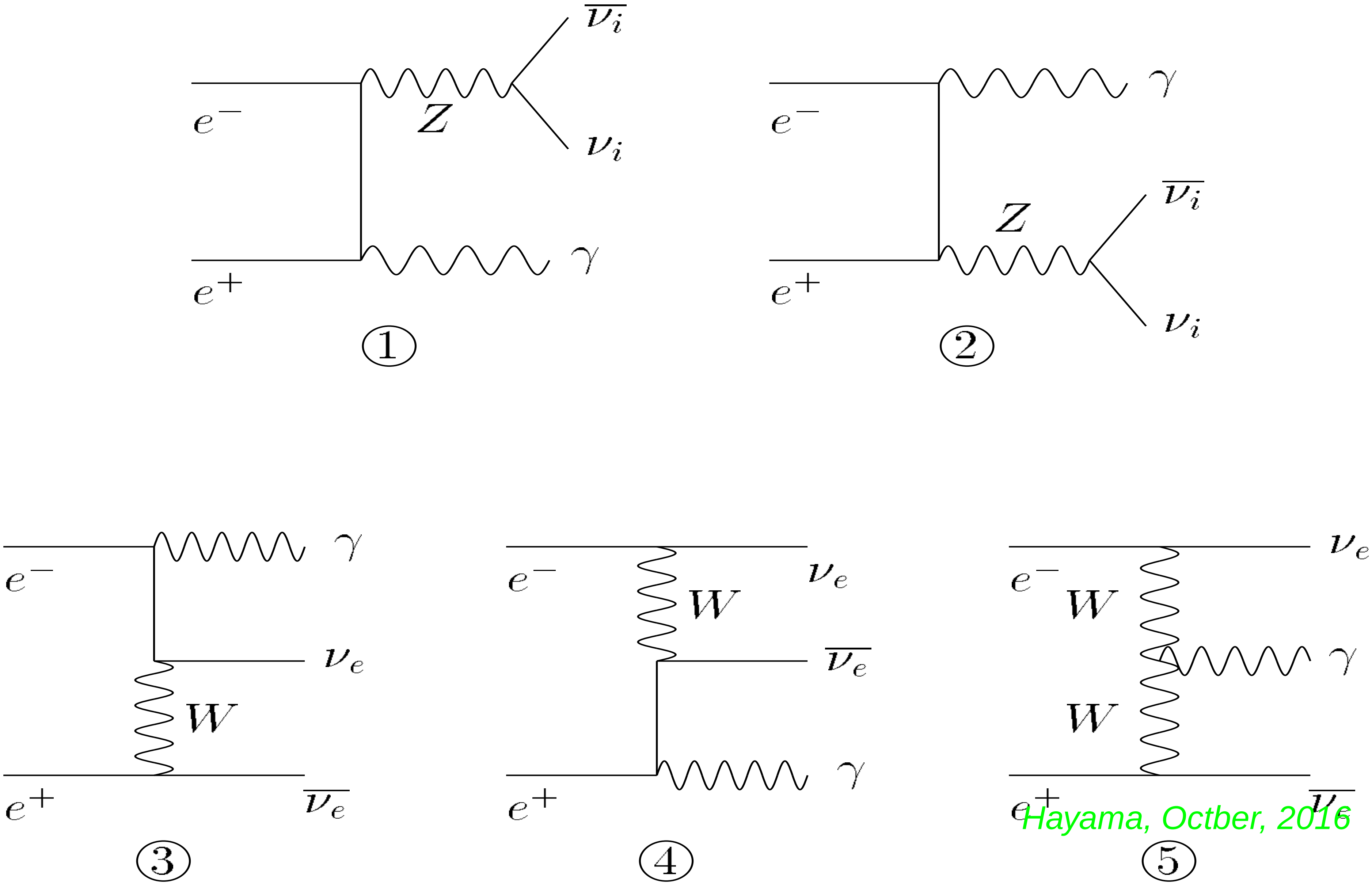
$$\Lambda_+(p, 0) = \tilde{\Lambda}_+(p, 0) = u(p,+)\bar{u}(p,+) + u(p,-)\bar{u}(p,-).$$

Similarly Λ_- can be expressed in terms of $v(p, \alpha)\bar{v}(p, \bar{\alpha})$, see Appendix A. In practice,

1994 Structure of spin amplitudes

- General idea: to identify in amplitudes, with the help of gauge invariance structures responsable later for phase-space enhancements: collinear-soft etc. This is fundamental, specially from the point of view of Monte Carlo algorithm construction.
- Discussions with **Shimizu-san** were important.
- Z. Was *Gauge invariance, infrared / collinear singularities and tree level matrix element for e+ e- —> nu(e) anti-nu(e) gamma gamma* Eur.Phys.J. C44 (2005) 489,
- A. van Hameren, Z. Was, *Gauge invariant sub-structures of tree-level double-emission exact QCD spin amplitudes*, Eur.Phys.J. C61 (2009) 33
- Also in this case algebraic manipulation mehods were providing the reference calculations, necessary to cross check results.
- I was not able to find patterns automatically, but algebraic progams were essential for checks.
- Only some of the patterns appear naturally. Feynman diagrams 1 and 2 combined (next slide) are the complete amplitude for $\nu_\mu \bar{\nu}_\mu$ production.

Figure 1: *The Feynman diagrams for* $e^+e^- \to \bar{\nu}_e\nu_e\gamma$.



1994 Structure of spin amplitudes

- The first two diagrams represent initial state QED bremsstrahlun amplitudes for $\nu_\mu \bar{\nu}_\mu$ pair production. It can be divided into parts, corresponding to β_0, β_1 of Yennie-Frautshi-Suura exponentiation.
- Can separation be expanded to other cases, to higher orders, to terms of different singularities/enhancements?
- The answer seem to be always yes.
- It is also important to observe that it extends to QCD, to scalar QED ...
- I will sketch step for the calulation of single photon emission.
- Slide 9 single photon emision in $e^+e^- \to \nu_e \bar{\nu}_e$
- Slide 10 double gluon emission in $q\bar{q} \to l^+l^-$

1994 Structure of spin amplitudes

$$
\begin{aligned}
&\mathcal{M}_{1\{I\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1}\right) = \mathcal{M}^0 + \mathcal{M}^1 + \mathcal{M}^2 + \mathcal{M}^3 \\
&\mathcal{M}^0 = eQ_e\;\bar v(p_b,\lambda_b)\;\mathbf{M}^{bd}_{\{I\}}\;\frac{\not p_a + m - \not k_1}{-2k_1p_a}\;\not\epsilon^\star_{\sigma_1}(k_1)\;u(p_a,\lambda_a) \\
&\;+ eQ_e\;\bar v(p_b,\lambda_b)\;\not\epsilon^\star_{\sigma_1}(k_1)\;\frac{-\not p_b + m + \not k_1}{-2k_1p_b}\;\mathbf{M}^{ac}_{\{I\}}\;u(p_a,\lambda_a) \\
&\mathcal{M}^1 = \mathcal{M}^{1'} + \mathcal{M}^{1''} \\
&\mathcal{M}^{1'} = +e\;\bar v(p_b,\lambda_b)\;\;\mathbf{M}^{bd,ac}_{\{I\}}\;u(p_a,\lambda_a)\epsilon^\star_{\sigma_1}(k_1)\cdot(p_c - p_a)\frac{1}{t_a - M_W^2}\,\frac{1}{t_b - M_W^2}, \\
&\mathcal{M}^{1''} = +e\;\bar v(p_b,\lambda_b)\;\;\mathbf{M}^{bd,ac}_{\{I\}}\;u(p_a,\lambda_a)\epsilon^\star_{\sigma_1}(k_1)\cdot(p_b - p_d)\frac{1}{t_a - M_W^2}\,\frac{1}{t_b - M_W^2}, \\
&\mathcal{M}^2 = +e\;\bar v(p_b,\lambda_b)g^{We\nu}_{\lambda_b,\lambda_d}\;\not\epsilon^\star_{\sigma_1}(k_1)\;\;v(p_d,\lambda_d)\bar u(p_c,\lambda_c)g^{We\nu}_{\lambda_c,\lambda_a}\;\not k_1\;\;u(p_a,\lambda_a)\frac{1}{t_a - M_W^2}\,\frac{1}{t_b - M_W^2} \\
&\mathcal{M}^3 = -e\;\bar v(p_b,\lambda_b)g^{We\nu}_{\lambda_b,\lambda_d}\;\not k_1\;\;v(p_d,\lambda_d)\bar u(p_c,\lambda_c)g^{We\nu}_{\lambda_c,\lambda_a}\;\not\epsilon^\star_{\sigma_1}(k_1)\;\;u(p_a,\lambda_a)\frac{1}{t_a - M_W^2}\,\frac{1}{t_b - M_W^2},
\end{aligned}
\tag{1}
$$

- Once manipulations completed, we separate the complete spin amplitude for the process $e^+e^- \to \bar\nu_e\nu_e\gamma$ into *six* individually QED gauge invariant parts. This conclusion is rather straightforward to check, replacing photon polarization vector with its four-momentum. Each of the obtained parts has well defined physical interpretation.
- It is also easy to verify that the gauge invariance of each part can be preserved to the case of the extrapolation, when because of additional photons, condition $p_a + p_b = p_c + p_d + k_1$ is not valid.

$$\mathcal{M}^{a,b} = \frac{1}{2}\,\bar{v}(p)\Big(T^aT^bI^{(1,2)} + T^bT^aI^{(2,1)}\Big)u(q)\;. \tag{2}$$

For the T^aT^b-part, we find

$$I^{(1,2)} = \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not e_1 \not k_1}{2p\cdot k_1}\right)\not J\left(\frac{\not k_2\not e_2}{2q\cdot k_2} + \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right) \tag{3}$$

$$+ \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not e_1\not k_1}{2p\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not e_2\not k_2}{2p\cdot k_2}\right)\not J \tag{4}$$

$$+\not J\frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not k_1\not e_1}{2q\cdot k_1}\right)\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not k_2\not e_2}{2q\cdot k_2}\right) \tag{5}$$

$$+\not J\left(1 - \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} - \frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\right)\left(\frac{k_1\cdot e_2}{k_1\cdot k_2}\frac{k_2\cdot e_1}{k_1\cdot k_2} - \frac{e_1\cdot e_2}{k_1\cdot k_2}\right) \tag{6}$$

$$-\frac{1}{4}\,\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{\not e_1\not k_1\not e_2\not k_2 - \not e_2\not k_2\not e_1\not k_1}{k_1\cdot k_2}\right)\not J \tag{7}$$

$$-\frac{1}{4}\not J\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{\not k_1\not e_1\not k_2\not e_2 - \not k_2\not e_2\not k_1\not e_1}{k_1\cdot k_2}\right)\,. \tag{8}$$

The part proportional to T^bT^a is obtained by a permutation of the momenta and polarization vectors of the gluons.

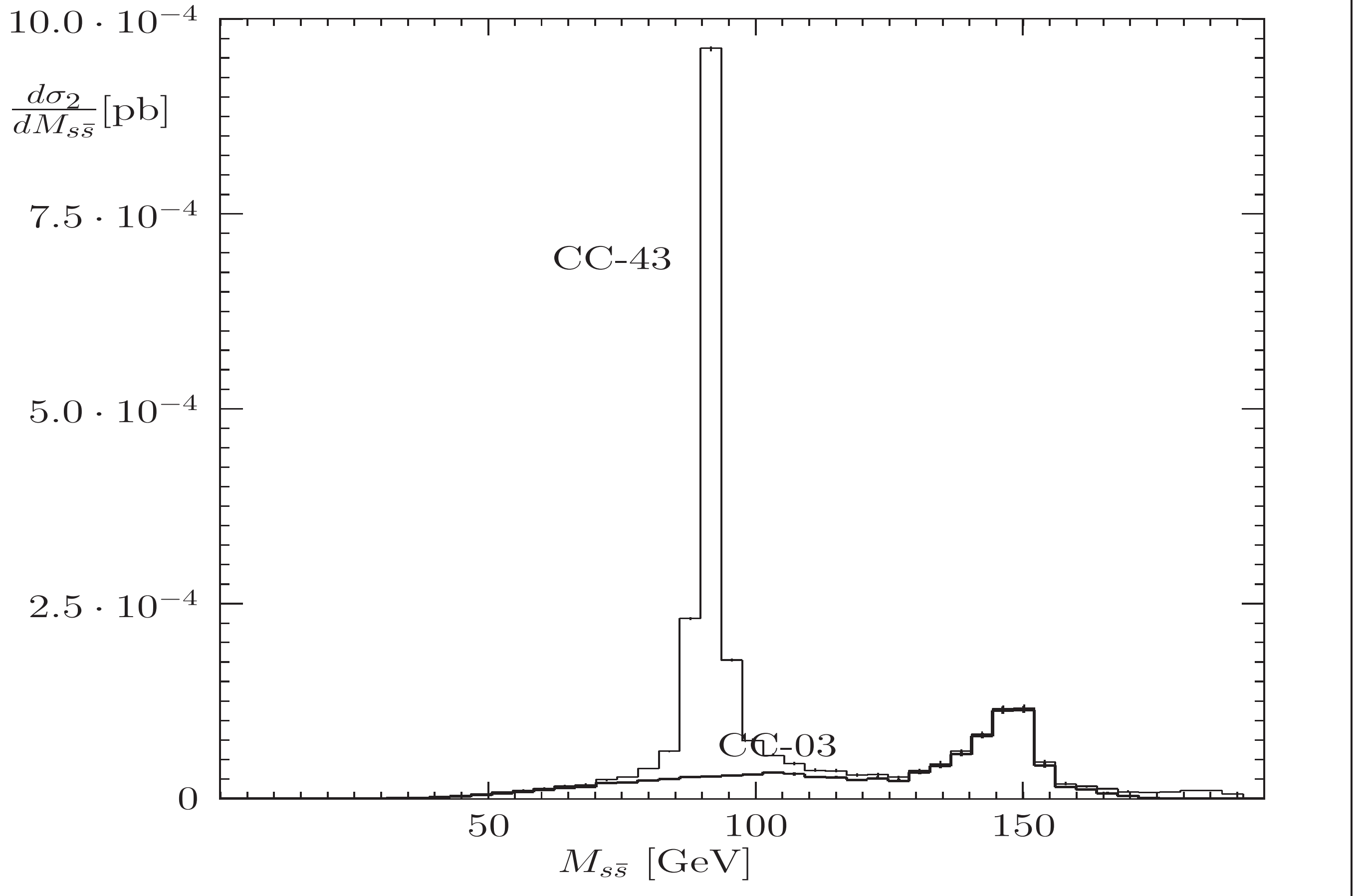
1998 Unexpected features,

- The main purpose of my 1996 visit at KEK in MinamiTateya group, was to work on Grace spin amplitudes (Comput.Phys.Commun. 153 (2003) 106).
- Our KORALW Monte Carlo used Grace spin amplitudes for the $e^+e^- \to 4$ fermion processes.
- Monte Carlo itegration of phase space regions where collinear configurations were present, resulted in numerical difficulties. Abnormal features appeared. This required careful and painful work to avoid ‘trivial’ mistakes. Kind of faked ‘New Physics’, phenomnenon.
- Let me show rather unexpected, at a time, example from the publication: *Four quark final state in W pair production: Case of signal and background*, T. Ishikawa, Y. Kurihara, M. Skrzypek, Z. Was, Eur. Phys. J. C4 (1998) 75.
- Interplay of theoretical effects and selection cuts can be confusing:

2→ 4 fermion processes at 195 GeV

W-pair production and decay, veto cut on 2 jets.

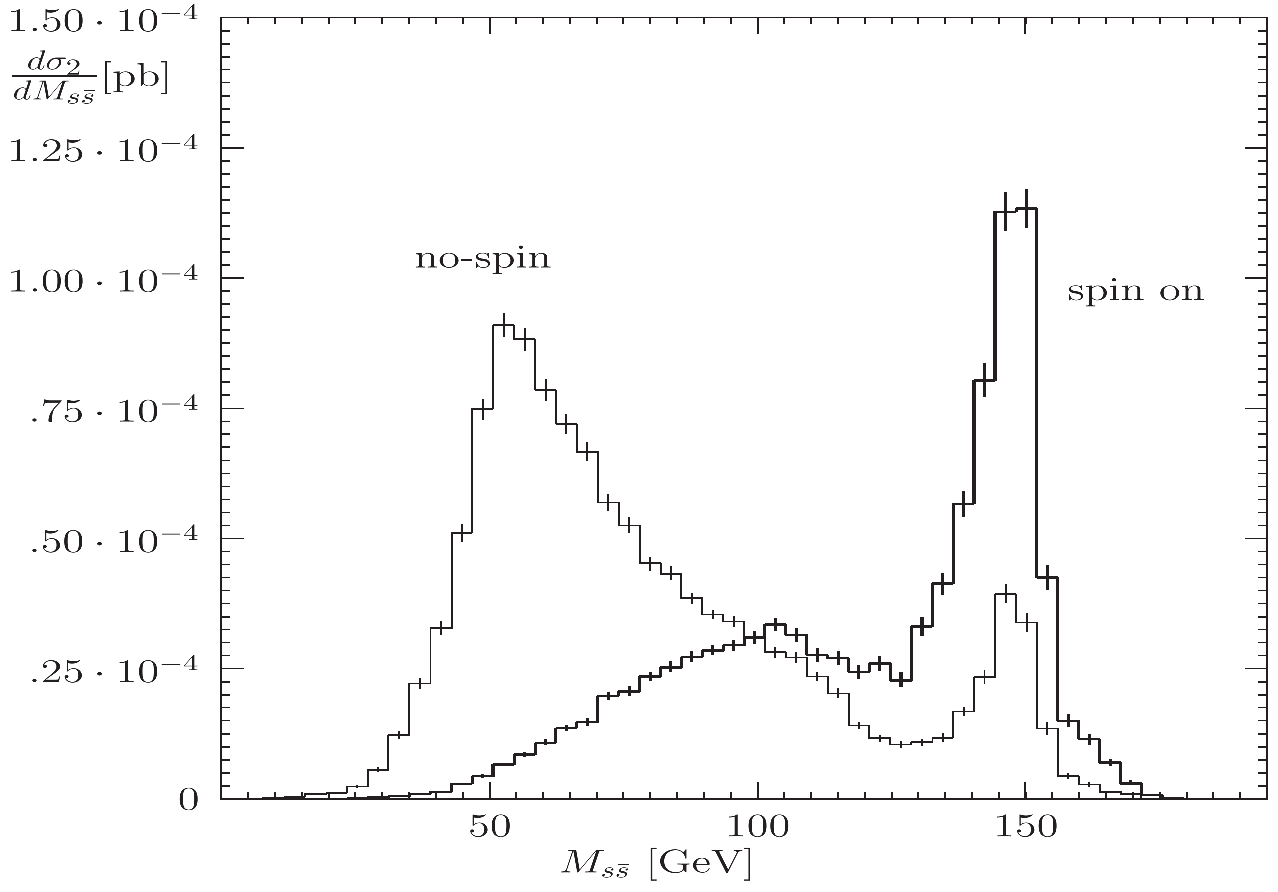
Figure 4: The $\frac{d\sigma_2}{dM_{s\bar{s}}}$ differential distribution of the "visible" $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 2: CC-03 (thick line); and type 4: CC-43 (thin line). See Appendices A, B for a complete definition of all input parameters.



$2 \to 4$ fermion processes at 195 GeV

effect due to spin: sometimes negigible sometimes not.

Figure 3: The $\frac{d\sigma_2}{dM_{s\bar{s}}}$ differential distribution of the "visible" $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 1: CC-03 no spin correlation (thin line); and type 2: CC-03 spin correlations switched on (thick line). See Appendices A, B for a complete definition of all input parameters.



Higgs parity in $H \to \tau\tau$

- I will show another example where complex observables need to be defined.
- Important is slide nr **29**: in case of $H \to \tau\tau$ $\tau \to 3\pi\nu$ we may want to mesaure simultaneously 4 or 16 angles.
- Each providing some of CP effect ...
- ...but all of them correlated and under pressure from backgrounds.
- These angles are extension of single acoplanarity angle which is used in case $H \to \tau^+\tau^-$ $\tau^\pm \to \pi^\pm\pi^0\nu$. Observable of mulidimensional nature cab be controlled with ML techniques.
- Risk of biasing.
- I will skip some slides of introduction, we have no time to present that.
-

The Higgs boson's parity

- H/A parity information can be extracted from the correlations between τ^+ and τ^- spin components which are further reflected in correlations between the τ decay products in the plane transverse to the $\tau^+\tau^-$ axes.
- The decay probability

$$\Gamma(H/A \to \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-}$$

  is sensitive to the $\tau^{\pm}$ polarization vectors s^{τ^-} and s^{τ^+} (defined in their respective rest frames). The symbols $\parallel, \perp$ denote components parallel/transverse to the Higgs boson momentum as seen from the respective $\tau^{\pm}$ rest frames.
- This spin case is technically easy, because 'Higgs spin' is blind on Higgs origin.
  Z. Was, M. Worek, Acta Phys. Polon. B33 (2002) 1875.

Formalism for $\tau^+\tau^-$: nothing changes

- Because narrow τ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

- This formalism is fine, but because of over 20 τ decay channels we have over 400 distinct processes. Also picture of production and decay are mixed.
- Below only τ spin indices are explicitly written:

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Cross section can be re-written into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

General formalism for semileptonic decays

- Matrix element used in TAUOLA for semileptonic decay

$$\tau(P,s) \to \nu_\tau(N) X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N)\gamma^\mu (v + a\gamma_5) u(P) J_\mu$$

- J_μ the current depends on the momenta of all hadrons

$$|\mathcal{M}|^2 = G^2 \frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^* \cdot N)J_\mu + (J \cdot N)J^*_\mu - (J^* \cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2} m_\nu M (J^* \cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2} m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma$$

Higgs Boson Parity

- *Decay probability in formalism of Kramer et al.*

$$\Gamma(H/A^0 \to \tau^+\tau^-) \sim 1 - s_\parallel^{\tau^+} s_\parallel^{\tau^-} \pm s_\perp^{\tau^+} s_\perp^{\tau^-}$$

- s^τ *is the* τ *polarization vectors.*
- $\parallel$ / $\perp$ *denote components parallel / transverse to the Higgs boson momentum.*
- *The spin weight is given by the following formula*

$$wt = \frac{1}{4}\left(1 + \sum_{ij=1}^{3} R_{ij} h^i h^j\right)$$

$$R_{33} = -1, \quad R_{11} = \pm 1, \quad R_{22} = \pm 1$$

- *Components for pure scalar and pseudoscalar Higgs boson respectively.*

General idea

Density matrix

Only transverse spin correlations between τ^+ and τ^- are different for scalar and pseudoscalar Higgs

- The correlations can not be measured directly
- One need to measure distributions of τ decay products
- Precisely their transverse (to τ direction in Higgs boson rest frame) momenta
- Most sensitive to spin is $\tau^\pm \to \pi^\pm \nu$
- The largest branching ratio (25 %) has $\tau^\pm \to \pi^\pm \pi^0 \nu$ and we can look on transverse spin correlations of $\rho^\pm \to \pi^\pm \pi^0$ decays.

Scalar or Pseudoscalar?

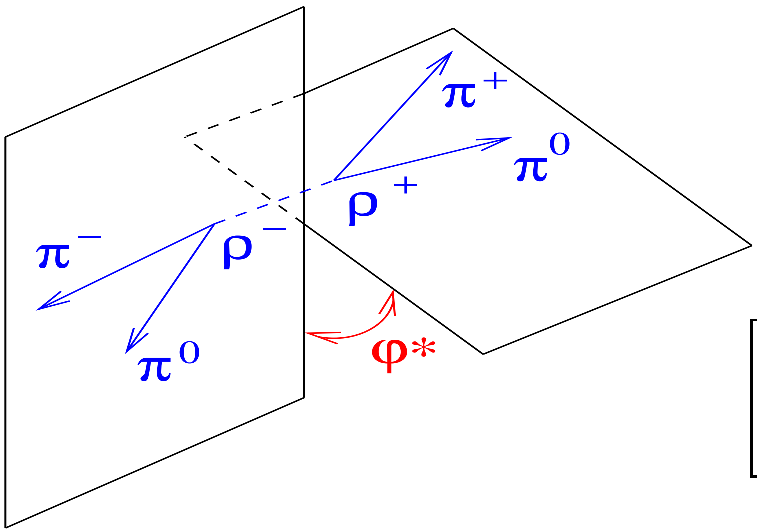
$\mathcal{P}$ure $\mathcal{S}$calar $\mathcal{A}$nd $\mathcal{P}$seudoscalar $\mathcal{H}$iggs $\mathcal{B}$oson

- *Case of* $\tau \to \rho\nu_\tau$ *decay,* $\mathcal{BR}(\tau \to \rho\nu_\tau) = 25\%$
- *The polarimeter vector is given by the formula where* q *for* $\pi^\pm - \pi^0$, N *for* ν_τ.

$$h^i = \mathcal{N}\Big(2(q \cdot N)q^i - q^2 N^i\Big)$$

$$q \cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau$$

- *Acoplanarity of* ρ^+ *and* ρ^- *decay prod. (in* $\rho^+\rho^-$ *r.f.) and events separation.*

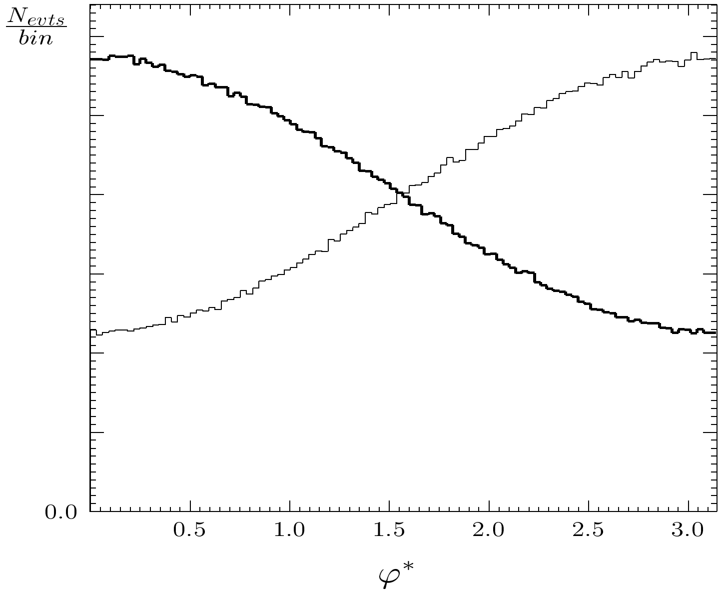


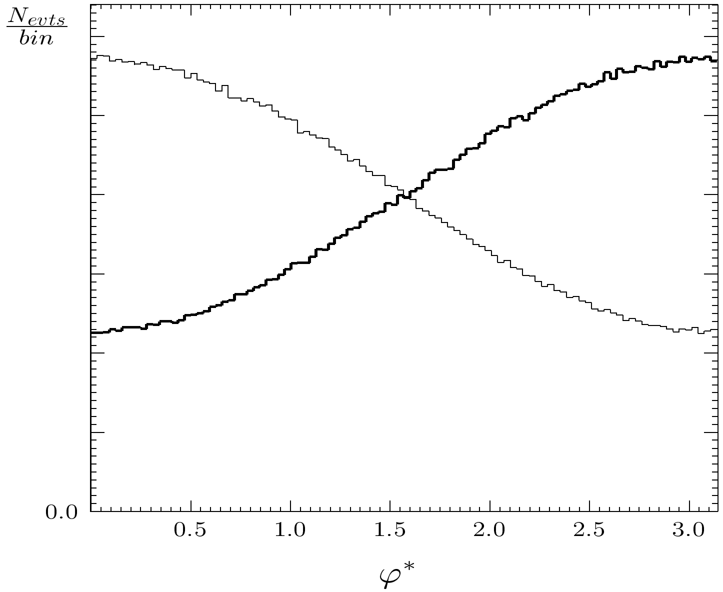
$y_1 y_2 > 0\,;\qquad y_1 y_2 < 0$ *(in* $\tau^\pm$ *r.f.'s)*

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}\,;\qquad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}.$$

Scalar or Pseudoscalar ?

Results Without Smearing





- *The $\rho^+\rho^-$ decay products' acoplanarity distribution without any smearing .*
- *Selection $y_1 y_2 > 0$ is used in the left plot, $y_1 y_2 < 0$ is used for the right plot.*
- *Thick line denote the case of the scalar Higgs and thin lines the pseudoscalar.*
- *Complete spin correlations of $h \to \tau^+\tau^-$, $\tau^\pm \to \rho^\pm\nu$, $\rho^\pm \to \pi^\pm\pi^0$ incl.*

Scalar or Pseudoscalar ?

Phenomenology Of General Case

- Higgs boson Yukawa coupling expresed with the help of the scalar–pseudoscalar mixing angle ϕ

$$\bar{\tau} N(\cos\phi + i\sin\phi\gamma_5)\tau$$

- *Decay probability for the mixed scalar–pseudoscalar case*

$$\Gamma(h_{mix} \to \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} + s_{\perp}^{\tau^+} R(2\phi)\, s_{\perp}^{\tau^-}$$

- $R(2\phi) -$ *operator for the rotation by angle* 2ϕ *around the* $\parallel$ *direction.*

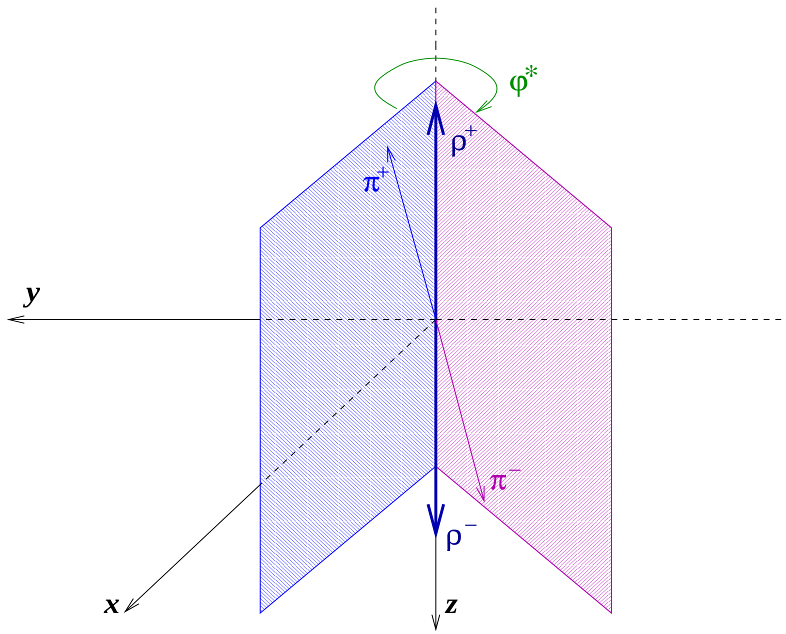
$$R_{11} = R_{22} = \cos 2\phi \qquad R_{12} = -R_{21} = \sin 2\phi$$

- *Pure scalar case is reproduced for* $\phi = 0$*.*
- *For* $\phi = \pi/2$ *we reproduce the pure pseudoscalar case.*

Scalar or Pseudoscalar ?

Observable For Mixed Scalar–Pseudoscalar Case

- *For mixing angle ϕ, transverse component of τ^+ spin polarization vector is correlated with the one of τ^- rotated by angle 2ϕ.*
- *Acoplanarity $0 < \varphi^* < 2\pi$ is of physical interest, not just $\arccos \mathbf{n}_- \cdot \mathbf{n}_+$.*
- *Distinguish between the two cases $0 < \varphi^* < \pi$ and $2\pi - \varphi^*$*
- *If no separation made the parity effect would wash itself out.*



Normal to planes: $\mathbf{n}_\pm = \mathbf{p}_{\pi^\pm} \times \mathbf{p}_{\pi^0}$

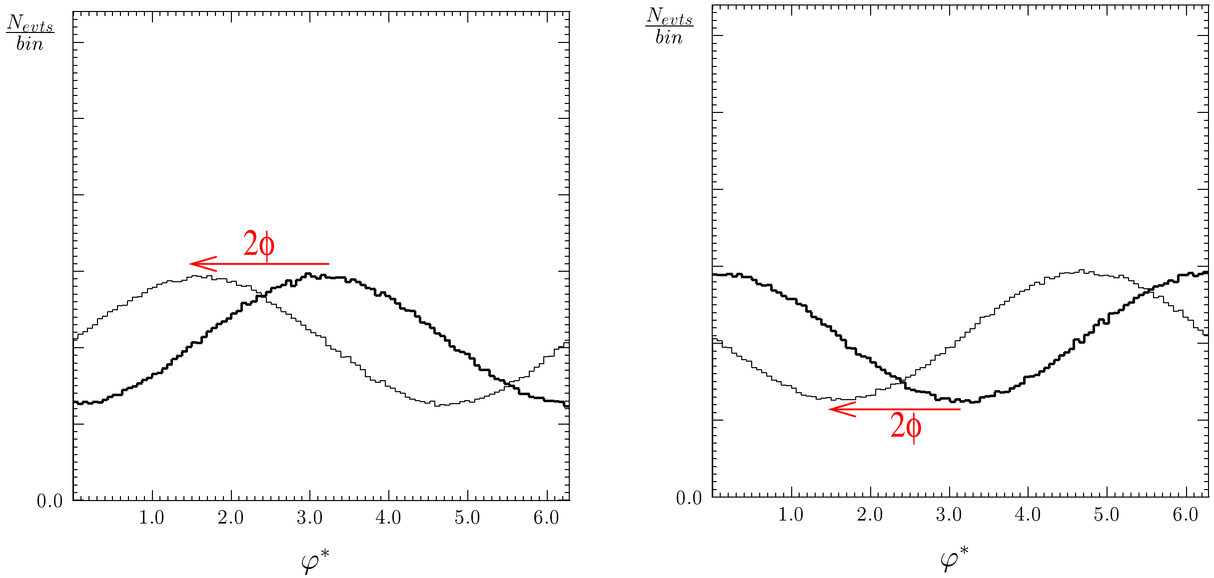
Find the sign of $\mathbf{p}_{\pi^-} \cdot \mathbf{n}_+$

Negative $0 < \varphi^* < \pi$

Otherwise $2\pi - \varphi^*$

Scalar or Pseudoscalar ?

Results For Mixed Scalar−Pseudoscalar Case



- *Only events where the signs of y1 and y2 are the same whether calculated using the method without or with the help of the τ impact parameter.*
- *Detector-like set-up is included (SIMDET).*
- *The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.*

Precision on ϕ $\sim$ 6 $^\circ$, for $1ab^{-1}$ and 350 GeV CMS.

Hadronic currents: source of th. uncertainty

- Improvements for ρ channel are technically straightforward: single real function to be fitted: $J^\mu = (p_{\pi^\pm} - p_{\pi^0})^\mu F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^\mu F_S(Q^2)$ ($F_S \simeq 0$).
- For 3-scalar states: 4 complex function 3 variables each. Role of theoretical assumptions is essential. Agreement on 1-dim distribution is a consistency check.
- No go for model independent measurements? Not necessarily. Use of all dimensions for data distributions: invariant masses Q^2, s_1, s_2 as arguments of form-factors. Angular asymmetries help to separate currents: scalar $J_4^\mu \sim Q^\mu = (p_1 + p_2 + p_3)^\mu$, vector $J_1^\mu \sim (p_1 - p_3)^\mu|_{\perp Q}$ and $J_2^\mu \sim (p_2 - p_3)^\mu|_{\perp Q}$ and finally pseudovector $J_5^\mu \sim \epsilon(\mu, p_1, p_2, p_3)$. Dependence on hadronic currents remain in calculation of polarimetric vectors.
- Model independent methods, template methods, neural networks, multidimensional signatures. It was easier for Cleo. There, τ's were produced nearly at rest, ν_τ four-momentum was easy to reconstruct.
- Fitting in complex situation is ... **well complex ! Instead of acoplanarity angle in $a_1 - a_1$ case we have 16 such angles.**

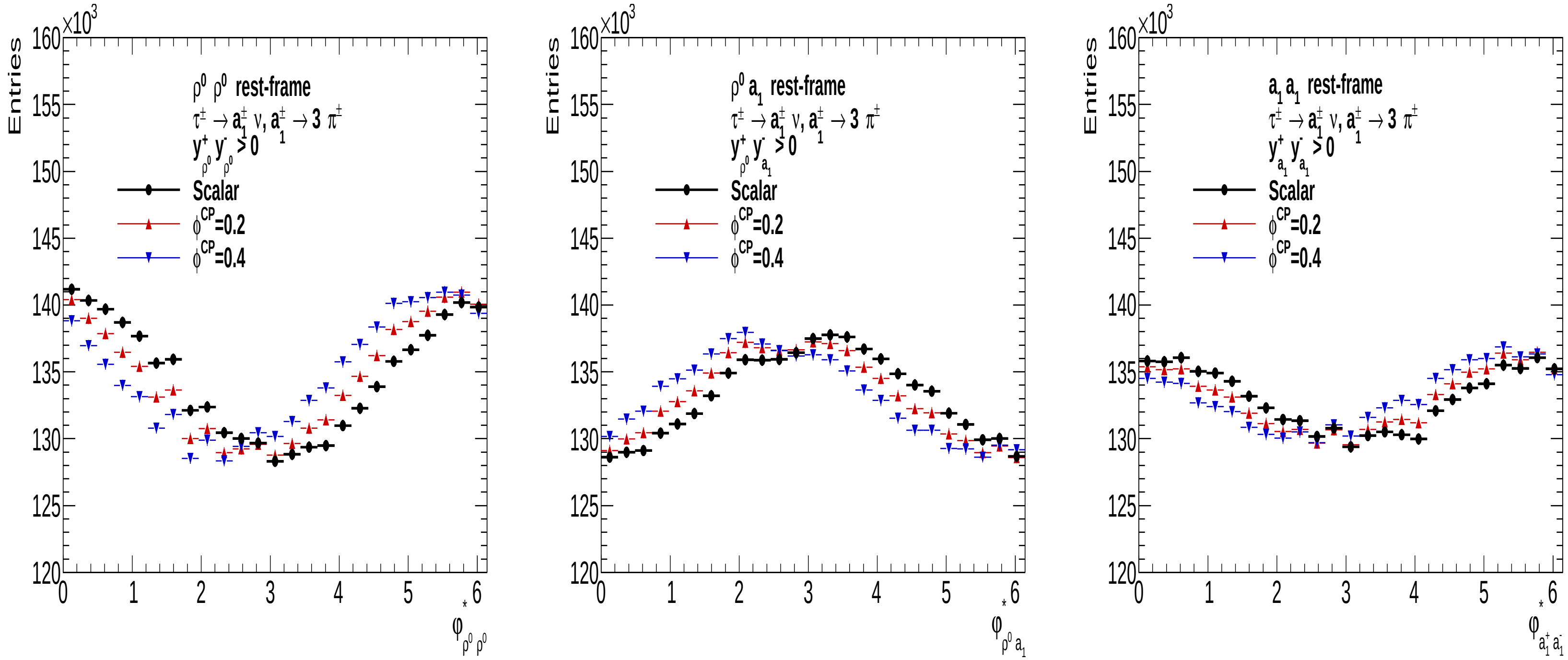
ML techiques are needed.

from $\rho^{\pm}$ to $a_1^{\pm}$ case.

1. In case of $\tau \to \rho\nu$ there was one decay plane to define and sign of CP sensitive sinusoid was dependent on sign of $y_+ y_-$.
2. In case of $\tau \to a_1\nu$ four planes can be defined. Two for $a_1 \to \pi\rho^0$ and another two for $\rho^0 \to \pi^+\pi^-$ decays.
3. We end up with 4 (or 16) angular distributions at number of y_i like variables.
4. That means meny sub categories to define sample ...
5. All distributions are correlated.
6. Methods of Maschine Learning necessary, to evaluate sensitivity of mult-dimensional signatures $\rightarrow$ Brian.

Neural Network for CP parity of Higgs,arXiv:1608.02609

Acoplanarity angles of oriented half decay planes: $\varphi^*_{\rho^0\rho^0}$ (left), $\varphi^*_{a_1\rho^0}$ (middle) and $\varphi^*_{a_1 a_1}$ (right), for events grouped by the sign of $y^+_{\rho^0} y^-_{\rho^0}$, $y^+_{a_1} y^-_{\rho^0}$ and $y^+_{a_1} y^-_{a_1}$ respectively. Three CP mixing angles ϕ^{CP} = 0.0 (scalar), 0.2 and 0.4. Note scale, effect on individual plot is so much smaller now. But up to **16 plots like that** have to be measured, correlations understood. But physics model depends on 1 parameter only and effect of ϕ^{CP}, the Higgs mixing scalar pseudoscalar angle, is always a linear shift.



Results relevant for fitting and for τ leptons.

1. *W production at LHC: lepton angular distributions and reference frames for probing hard QCD*, E. Richter-Was and Z. Was, arXiv:1609.02536
2. *Potential for optimizing Higgs boson CP measurement in H to tau tau decay at LHC and ML techniques* , R. Jozefowicz, E. Richter-Was and Z. Was,arXiv:1608.02609
3. *Separating electroweak and strong interactions in Drell—Yan processes at LHC: leptons angular distributions and reference frames*, E. Richter-Was and Z. Was, Eur.Phys.J. C76 (2016) 473
4. *“. Production of tau lepton pairs with high pT jets at the LHC and the TauSpinner reweighting algorithm”*, J. Kalinowski, W. Kotlarski, E. Richter-Was and Z. Was, arXiv:1604.00964
5. *“TauSpinner Program for Studies on Spin Effect in tau Production at the LHC”*, Z. Czyczula, T. Przedzinski and Z. Was, Eur. Phys. J. C **72**, 1988 (2012)

Warning message

- Result depend on model assumptions. Models inspired with results ...
  Fitting setup $\rightarrow$ biases.
- Our algorithms are far less elaborate than human eye/brain.
- That may look worrisome.

- Biases in art, Giuseppe Arcimboldo (1572 - 1593).

We are not alone with the problem

Figure 2: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don't.

From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

Pattern recognition is an active field and deep concern and not only for us.

Summary

- I have presented scattered results where use of computer algebraic methods or pattern recognition techniques (Mashine Learning) was necessary.
- My experience with such approaches started in 1996 in MinamiTateya group.
- Working on my talk was inspiring to myself. Also, it was not easy to select slides for a coherent presentation.
- In fact, I am not sure if I was able to send the message: computer algebra methods $\longrightarrow$ correct huge expressions $\longrightarrow$ loss of control on what should come out $\longrightarrow$ how to understand/interpret/use $\longrightarrow$ we are not alone with such difficulties $\longrightarrow$ how to avoid detection of non-existent...
- Manpower/training is an essential issue for continuity of projects.
- **The challenges are more for newcomers, who may have missed long years of rewarding failures.**