Automated calculation of matrix elements and physics motivated observables

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- (1) Once computers arrived, for me it was year 1980, approach to phenomenology of theory/model based predictions could change a lot.

- (2) Numerous benefits became available. Drawbacks appeared as well. For example, methods of special functions expansions seem to be not as widespread as in the past.

- (3) I will concentrate on examples of my personal experience. I do not have any intensions to be systematic and balanced. Better picture will hopefully appear from other talks, e.g. examples of special functions expansions.

- (4) I will not focus on successes of the field. These are well known.

- (5) I will review traps which turned out to be rewarding to me once resolved; often in an unexpected way.

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Hayama, October, 2016
• Encouraged by Simizu-sensei conference, I choose to say what I always wanted, but never did.

• I thought the talk will be easy to prepare....

• In contrary, I found work frustrating, but rewarding.

• My plan is to show several simple examples of challenges resulting from complexity and how automated calculations were of help, but also a source of difficulties.

• Older examples originate from my work in Shimizu-san Minami Tateya group I visited in 1995.

• Each example in principle require substantial introduction, impossible to cover in one talk.

• My slides will show, outcome of my crippled attempts.

Z. Was Hayama, October, 2016
At that time Poland was an isolated place, but with enormous in-flow of students to research. In reality a lot of contacts existed, but it was not to be seen by me.

Access to computing was limited and in fact quite awkward: hopeless loss of time it seemed.

One of my first project was to evaluate spin density matrix for the process \( e^+ e^- \rightarrow \tau^+ \tau^- \gamma \) at Petra/PEP energies. Monte Carlo Simulation of the Process \( e^+ e^- \rightarrow \tau^+ \tau^- \gamma \) at Petra/PEP energies. Including Radiative O(\(\alpha^3\)) QED Corrections, Mass and Spin S. Jadach, Z. Was (Jagiellonian U.). Mar 1984. Comput.Phys.Commun. 36 (1985) 191.

This work was performed under guidance of Prof. S. Jadach.

Fantastic experience in looking at spin amplitudes as (reducible) representations of (Lorentz × gauge) groups.

It was great that we could spend all necessary time to understand details of what we were doing.

In this particular case, how to represent moderately complicated formulas of spin states into compact forms, exploiting geometrical properties of formulae.
To simplify and to understand amplitudes:
Compact and intuitive representations of $\tau^+ \tau^-$ spin density matrix

In the last formula the index $i = 1, 2, 3$ numbers the three components of $w_i$ in the rest frame of the $\tau^+$ lepton and the $k = 1, 2, 3$ numbers the axes in the rest system of the $\tau^-$ lepton. In both rest frames the third axis is the spin quantisation axis as in the definition of $\alpha_1$ and $\alpha_2$ and the first axis is defined to be perpendicular to the reaction plane i.e. along $\tau$-vector. In (2.5) the absence of terms linear in $w_k$ like $\sum \frac{R_{ik}^0}{2} w_k^2$ and $\sum \frac{R_{ik}^0}{2} w_i^2$ means that each $\tau^\pm$ separately is not polarized in the lowest order. There are, however, correlations between $w_1$ and $w_2$ which are controlled by the matrix $R_{ik}^0$. We extend the matrix $R_{ik}^0$ to $R_{ab}^0$ with $a, b = 0, 1, 2, 3$ obtaining

$$R_{ab}^0 = \begin{bmatrix}
1 + c^2 + M^2 s^2, & 0, & 0, & 0 \\
0, & -(1 - M^2) s^2, & 0, & 0 \\
0, & 0, & (1 + M^2) s^2, & 2Mc s \\
0, & 0, & 2Mc s, & 1 + c^2 - M^2 s^2
\end{bmatrix}.$$ (2.6)

In order to calculate the matrix $R_{ab}^0$ as given by equation (2.6) directly from our spin amplitudes defined in Eq. (2.3), we have to translate the bispinor indices in the joint density matrix given by

$$Q_{a_1a_2b_1b_2}^0 = \frac{1}{2} \sum_{\lambda_1\lambda_2} M_{12}^0 (M_{\lambda_1\lambda_2}^0)^*$$

$$= \frac{1}{2} U^2 \left[ |A_+ A_+| + A_+ A_- c^2 + M^2 s^2 A_- A_- - \frac{i}{2} (A_+ A_- - A_- A_+) 2Mc s \right]$$ (2.7)

into vector indices $a$ and $b$, see (2.6). The answer may be read off from Eq. (2.4) by substituting in the operator $A_\pm(p, w)$ as polarisation vectors the three space-like vectors $\vec{e}_1 = (0, 1, 0, 0)$, $\vec{e}_2 = (0, 0, 1, 0)$ and $\vec{e}_3 = (0, 0, 0, 1)$ and comparing the results with the bispinor quantities $u(p, \alpha)u(p, \alpha)$ in the $\tau$ rest frame, $p = (M, 0, 0, 0)$, $\alpha$ being the spin projection onto $\vec{e}_3$. The result is

$$A_+(p, \vec{e}_1) - A_+(p, 0) = \tilde{A}_+(p, \vec{e}_1) = u(p, +)u(p, -) + u(p, -)u(p, +),$$

$$A_+(p, \vec{e}_2) - A_+(p, 0) = \tilde{A}_+(p, \vec{e}_2) = iu(p, -)u(p, +) - iu(p, +)u(p, -),$$

$$A_+(p, \vec{e}_3) - A_+(p, 0) = \tilde{A}_+(p, \vec{e}_3) = u(p, +)u(p, -) - u(p, -)u(p, +).$$ (2.8)

and in addition

$$A_+(p, 0) = \tilde{A}_+(p, 0) = u(p, +)u(p, +) + u(p, -)u(p, -).$$

Similarly $A_-$ can be expressed in terms of $v\tilde{v}$, see Appendix A. In practice,
1994 Structure of spin amplitudes

- General idea: to identify in amplitudes, with the help of gauge invariance structures responsible later for phase-space enhancements: collinear-soft etc. This is fundamental, specially from the point of view of Monte Carlo algorithm construction.

- Discussions with Shimizu-san were important.

- Z. Was *Gauge invariance, infrared / collinear singularities and tree level matrix element for e+ e- \(\rightarrow\) nu(e) anti-nu(e) gamma gamma* Eur.Phys.J. C44 (2005) 489,


- Also in this case algebraic manipulation methods were providing the reference calculations, necessary to cross check results.

- I was not able to find patterns automatically, but algebraic programs were essential for checks.

- Only some of the patterns appear naturally. Feynman diagrams 1 and 2 combined (next slide) are the complete amplitude for \(\nu_\mu \bar{\nu}_\mu\) production.
1994 Structure of spin amplitudes

Figure 1: The Feynman diagrams for $e^+e^- \rightarrow \bar{\nu}_e \nu_e \gamma$. 

1. $e^- \rightarrow \nu_i \rightarrow Z \rightarrow \bar{\nu}_i \rightarrow e^+$

2. $e^- \rightarrow \nu_i \rightarrow Z \rightarrow \bar{\nu}_i \rightarrow e^+$

3. $e^- \rightarrow \nu_e \rightarrow W \rightarrow \bar{\nu}_e \rightarrow e^+$

4. $e^- \rightarrow \nu_e \rightarrow W \rightarrow \bar{\nu}_e \rightarrow e^+$

5. $e^- \rightarrow W \rightarrow \nu_e \rightarrow \bar{\nu}_e \rightarrow \gamma$
The first two diagrams represent initial state QED bremsstrahlung amplitudes for $\nu_\mu \bar{\nu}_\mu$ pair production. It can be divided into parts, corresponding to $\beta_0, \beta_1$ of Yennie-Frautshi-Suura exponentiation.

Can separation be expanded to other cases, to higher orders, to terms of different singularities/enhancements?

The answer seem to be always yes.

It is also important to observe that it extends to QCD, to scalar QED ...

I will sketch step for the calculation of single photon emission.

Slide 9 single photon emission in $e^+e^- \to \nu_e \bar{\nu}_e$

Slide 10 double gluon emission in $q\bar{q} \to l^+l^-$
\[ M_{1\{I\}} \left( p^{k_1}_{\lambda \sigma_1} \right) = M^0 + M^1 + M^2 + M^3 \]

\[ M^0 = eQ_e \bar{\nu}(p_b, \lambda_b) M^{bd}_{\{I\}} \frac{\not{p}_a + m - \not{k}_1}{-2k_1p_a} \epsilon^*_\sigma_1(k_1) u(p_a, \lambda_a) \]

\[ + eQ_e \bar{\nu}(p_b, \lambda_b) \epsilon^*_\sigma_1(k_1) \frac{-\not{p}_b + m + \not{k}_1}{-2k_1p_b} M^{ac}_{\{I\}} u(p_a, \lambda_a) \]

\[ M^1 = M^{1'} + M^{1''} \]

\[ M^{1'} = + e \bar{\nu}(p_b, \lambda_b) M^{bd,ac}_{\{I\}} u(p_a, \lambda_a) \epsilon^*_\sigma_1(k_1) \cdot (p_c - p_a) \frac{1}{t_a - M^2_W} \frac{1}{t_b - M^2_W} \]

\[ M^{1''} = + e \bar{\nu}(p_b, \lambda_b) M^{bd,ac}_{\{I\}} u(p_a, \lambda_a) \epsilon^*_\sigma_1(k_1) \cdot (p_b - p_d) \frac{1}{t_a - M^2_W} \frac{1}{t_b - M^2_W} \]

\[ M^2 = + e \bar{\nu}(p_b, \lambda_b) g^{W\nu}_{\lambda_b, \lambda_d} \epsilon^*_\sigma_1(k_1) \nu(p_d, \lambda_d) \bar{u}(p_c, \lambda_c) g^{W\nu}_{\lambda_c, \lambda_a} k_1 u(p_a, \lambda_a) \frac{1}{t_a - M^2_W} \frac{1}{t_b - M^2_W} \]

\[ M^3 = - e \bar{\nu}(p_b, \lambda_b) g^{W\nu}_{\lambda_b, \lambda_d} k_1 \nu(p_d, \lambda_d) \bar{u}(p_c, \lambda_c) g^{W\nu}_{\lambda_c, \lambda_a} \epsilon^*_\sigma_1(k_1) u(p_a, \lambda_a) \frac{1}{t_a - M^2_W} \frac{1}{t_b - M^2_W} \]

- Once manipulations completed, we separate the complete spin amplitude for the process \( e^+ e^- \rightarrow \bar{\nu}_e \nu_e \gamma \)
  into six individually QED gauge invariant parts. This conclusion is rather straightforward to check, replacing photon polarization vector with its four-momentum. Each of the obtained parts has well defined physical interpretation.

- It is also easy to verify that the gauge invariance of each part can be preserved to the case of the extrapolation, when because of additional photons, condition \( p_a + p_b = p_c + p_d + k_1 \) is not valid.
$\mathcal{M}^{a,b} = \frac{1}{2} \bar{v}(p) \left( T^a T^b I^{(1,2)} + T^b T^a I^{(2,1)} \right) u(q)$.

For the $T^a T^b$-part, we find

\begin{align}
I^{(1,2)} &= \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{k_1 k_2}{2p \cdot k_1} \right) J \left( \frac{k_2 k_2}{2q \cdot k_2} + \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{q \cdot e_2}{q \cdot k_2} \right) \\
&+ \frac{p \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{k_1 k_2}{2p \cdot k_1} \right) \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{k_2 k_2}{2p \cdot k_2} \right) J \\
&+ J \left( \frac{q \cdot k_1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \left( \frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{k_1 k_2}{2q \cdot k_1} \right) \left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{k_2 k_2}{2q \cdot k_2} \right) \\
&+ J \left( 1 - \frac{p \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} - \frac{q \cdot k_1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{k_1 \cdot e_2}{k_1 \cdot k_2} \frac{k_2 \cdot e_1}{k_1 \cdot k_2} - \frac{e_1 \cdot e_2}{k_1 \cdot k_2} \right) \right) \\
&- \frac{1}{4} \frac{1}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \left( \frac{k_1 k_2 k_2 k_2 - k_2 k_2 k_2 k_2}{k_1 \cdot k_2} \right) J \\
&- \frac{1}{4} J \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \left( \frac{k_1 k_2 k_2 k_2 - k_2 k_2 k_2 k_2}{k_1 \cdot k_2} \right)
\end{align}

The part proportional to $T^b T^a$ is obtained by a permutation of the momenta and polarization vectors of the gluons.
1998 Unexpected features,

- The main purpose of my 1996 visit at KEK in MinamiTateya group, was to work on Grace spin amplitudes (Comput.Phys.Commun. 153 (2003) 106).

- Our KORALW Monte Carlo used Grace spin amplitudes for the $e^+e^- \rightarrow 4$ fermion processes.

- Monte Carlo integration of phase space regions where collinear configurations were present, resulted in numerical difficulties. Abnormal features appeared. This required careful and painful work to avoid ‘trivial’ mistakes. Kind of faked ‘New Physics’, phenomenon.


- Interplay of theoretical effects and selection cuts can be confusing:

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W-pair production and decay, veto cut on 2 jets.

Figure 4: The $\frac{d\sigma}{dM_{ss}}$ differential distribution of the “visible” $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 2: CC-03 (thick line); and type 4: CC-43 (thin line). See Appendices A, B for a complete definition of all input parameters.
Figure 3: The $\frac{d\sigma}{dM_{s\bar{s}}}$ differential distribution of the “visible” $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 1: CC-03 no spin correlation (thin line); and type 2: CC-03 spin correlations switched on (thick line). See Appendices A, B for a complete definition of all input parameters.
Higgs parity in $H \rightarrow \tau\tau$

- I will show another example where complex observables need to be defined.
- Important is slide nr 29: in case of $H \rightarrow \tau\tau \rightarrow 3\pi\nu$ we may want to measure simultaneously 4 or 16 angles.
- Each providing some of CP effect ...
- ...but all of them correlated and under pressure from backgrounds.
- These angles are extension of single acoplanarity angle which is used in case $H \rightarrow \tau^+\tau^- \tau^{\pm} \rightarrow \pi^{\pm}\pi^0\nu$. Observable of multidimensional nature can be controlled with ML techniques.
- Risk of biasing.
- I will skip some slides of introduction, we have no time to present that.
• $H/A$ parity information can be extracted from the correlations between $\tau^+$ and $\tau^-$ spin components which are further reflected in correlations between the $\tau$ decay products in the plane transverse to the $\tau^+\tau^-$ axes.

• The decay probability

$$\Gamma(H/A \to \tau^+\tau^-) \sim 1 - s_{||}^{\tau^+} s_{||}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-}$$

is sensitive to the $\tau^\pm$ polarization vectors $s_{||}^{\tau^\pm}$ and $s_{\perp}^{\tau^\pm}$ (defined in their respective rest frames). The symbols $||, \perp$ denote components parallel/transverse to the Higgs boson momentum as seen from the respective $\tau^\pm$ rest frames.

• This spin case is technically easy, because 'Higgs spin' is blind on Higgs origin.

General formula for tau production and decay.

**Formalism for $\tau^+\tau^-$**: nothing changes

- Because narrow $\tau$ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

  $$d\sigma = \sum_{\text{spin}} |M|^2 d\Omega = \sum_{\text{spin}} |M|^2 d\Omega_{\text{prod}} d\Omega_{\tau^+} d\Omega_{\tau^-}$$

- This formalism is fine, but because of over 20 $\tau$ decay channels we have over 400 distinct processes. Also picture of production and decay are mixed.

- Below only $\tau$ spin indices are explicitly written:

  $$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^2 \mathcal{M}_{\lambda_1\lambda_2}^{\text{prod}} \mathcal{M}_{\lambda_1}^{\tau^+} \mathcal{M}_{\lambda_2}^{\tau^-}$$

- Cross section can be re-written into **core formula of spin algorithms**

  $$d\sigma = \left(\sum_{\text{spin}} |M^{\text{prod}}|^2\right) \left(\sum_{\text{spin}} |M^{\tau^+}|^2\right) \left(\sum_{\text{spin}} |M^{\tau^-}|^2\right) wt\ d\Omega_{\text{prod}} d\Omega_{\tau^+} d\Omega_{\tau^-}$$

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\[ \frac{\alpha_{\text{QED}}}{\pi} \approx 0.2\% \text{ precision level} \]

**General formalism for semileptonic decays**

- Matrix element used in TAUOLA for semileptonic decay

\[
\tau(P, s) \rightarrow \nu_{\tau}(N) X \\
\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N) \gamma^\mu (v + a\gamma_5) u(P) J_\mu
\]

- \( J_\mu \) the current depends on the momenta of all hadrons

\[
|\mathcal{M}|^2 = G^2 \frac{v^2 + a^2}{2} (\omega + H_\mu s^\mu) \\
\omega = P^\mu (\Pi^\mu - \gamma v_\alpha \Pi^5_\mu) \\
H_\mu = \frac{1}{M} (M^2 \delta^\nu_\mu - P^\nu P^\mu) (\Pi^5_\nu - \gamma v_\alpha \Pi_\nu) \\
\Pi_\mu = 2[(J^* \cdot N) J_\mu + (J \cdot N) J^*_\mu - (J^* \cdot J) N_\mu] \\
\Pi^{5\mu} = 2 \text{Im} \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma \\
\gamma v_\alpha = -\frac{2v_\alpha}{v^2 + a^2} \\
\hat{\omega} = 2 \frac{v^2 - a^2}{v^2 + a^2} m_\nu M (J^* \cdot J) \\
\hat{H}^\mu = -2 \frac{v^2 - a^2}{v^2 + a^2} m_\nu \text{Im} \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma
\]
\[ \frac{\alpha_{QED}}{\pi} \approx 0.2\% \] precision level

**Higgs Boson Parity**

- **Decay probability in formalism of Kramer et al.**

\[ \Gamma(H/A^0 \rightarrow \tau^+ \tau^-) \sim 1 - s^+_\parallel s^-_\parallel \pm s^+_\perp s^-_\perp \]

- \( s^\tau \) is the \( \tau \) polarization vectors.

- \( \parallel / \perp \) denote components parallel / transverse to the Higgs boson momentum.

- The spin weight is given by the following formula

\[ wt = \frac{1}{4} \left( 1 + \sum_{i,j=1}^{3} R_{ij} h^i h^j \right) \]

\[ R_{33} = -1, \quad R_{11} = \pm 1, \quad R_{22} = \pm 1 \]

- Components for pure scalar and pseudoscalar Higgs boson respectively.

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Only transverse spin correlations between $\tau^+$ and $\tau^-$ are different for scalar and pseudoscalar Higgs

- The correlations can not be measured directly
- One need to measure distributions of $\tau$ decay products
- Precisely their transverse (to $\tau$ direction in Higgs boson rest frame) momenta
- Most sensitive to spin is $\tau^\pm \rightarrow \pi^\pm \nu$
- The largest branching ratio (25 %) has $\tau^\pm \rightarrow \pi^\pm \pi^0 \nu$ and we can look on transverse spin correlations of $\rho^\pm \rightarrow \pi^\pm \pi^0$ decays.
Scalar or Pseudoscalar?

**Pure Scalar And Pseudoscalar Higgs Boson**

- **Case of** \( \tau \to \rho \nu_\tau \) decay, \( \mathcal{BR}(\tau \to \rho \nu_\tau) = 25\% \)

- **The polarimeter vector** is given by the formula where \( q \) for \( \pi^\pm - \pi^0 \), \( N \) for \( \nu_\tau \).

\[
h^i = \mathcal{N} \left( 2(q \cdot N)q^i - q^2 N^i \right)
\]

\[
q \cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau
\]

- **Acoplanarity of** \( \rho^+ \) and \( \rho^- \) decay prod. (in \( \rho^+ \rho^- \) r.f.) and events separation.

\[
y_1 y_2 > 0 ; \quad y_1 y_2 < 0 \ (in \ \tau^\pm \ r.f.'s)
\]

\[
y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} ; \quad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}.
\]
• The $\rho^+ \rho^-$ decay products' acoplanarity distribution without any smearing.

• Selection $y_1 y_2 > 0$ is used in the left plot, $y_1 y_2 < 0$ is used for the right plot.

• Thick line denote the case of the scalar Higgs and thin lines the pseudoscalar.

• Complete spin correlations of $h \to \tau^+ \tau^-$, $\tau^\pm \to \rho^\pm \nu$, $\rho^\pm \to \pi^\pm \pi^0$ incl.

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Scalar or Pseudoscalar?

**Phenomenology Of General Case**

- Higgs boson Yukawa coupling expressed with the help of the scalar–pseudoscalar mixing angle $\phi$

$$\bar{\tau}N(\cos \phi + i \sin \phi \gamma_5)\tau$$

- Decay probability for the mixed scalar–pseudoscalar case

$$\Gamma(h_{mix} \rightarrow \tau^+ \tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} + s_{\perp}^{\tau^+} R(2\phi) s_{\perp}^{\tau^-}$$

- $R(2\phi)$ — operator for the rotation by angle $2\phi$ around the $\parallel$ direction.

$$R_{11} = R_{22} = \cos 2\phi \quad R_{12} = -R_{21} = \sin 2\phi$$

- Pure scalar case is reproduced for $\phi = 0$.

- For $\phi = \pi/2$ we reproduce the pure pseudoscalar case.
Scalar or Pseudoscalar?

Observable For Mixed Scalar–Pseudoscalar Case

- For mixing angle $\phi$, transverse component of $\tau^+$ spin polarization vector is correlated with the one of $\tau^-$ rotated by angle $2\phi$.

- Acoplanarity $0 < \varphi^* < 2\pi$ is of physical interest, not just $\arccos n_- \cdot n_+$.

- Distinguish between the two cases $0 < \varphi^* < \pi$ and $2\pi - \varphi^*$.

- If no separation made the parity effect would wash itself out.

Normal to planes: $n_\pm = p_{\pi\pm} \times p_{\pi^0}$

Find the sign of $p_{\pi^-} \cdot n_+$

Negative $0 < \varphi^* < \pi$

Otherwise $2\pi - \varphi^*$
• Only events where the signs of \( y_1 \) and \( y_2 \) are the same whether calculated using the method without or with the help of the \( \tau \) impact parameter.

• Detector-like set-up is included (SIMDET).

• The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.

Precision on \( \phi \sim 6^\circ \), for \( 1\text{ab}^{-1} \) and 350 GeV CMS.
Hadronic currents: source of th. uncertainty

• Improvements for $\rho$ channel are technically straightforward: single real function to be fitted: $J^{\mu} = (p_{\pi^\pm} - p_{\pi^0})^{\mu} F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^{\mu} F_S(Q^2) (F_S \simeq 0)$.

• For 3-scalar states: 4 complex function 3 variables each. Role of theoretical assumptions is essential. Agreement on 1-dim distribution is a consistency check.

• No go for model independent measurements? Not necessarily. Use of all dimensions for data distributions: invariant masses $Q^2$, $s_1$, $s_2$ as arguments of form-factors. Angular asymmetries help to separate currents: scalar $J_4^{\mu} \sim Q^{\mu} = (p_1 + p_2 + p_3)^{\mu}$, vector $J_1^{\mu} \sim (p_1 - p_3)^{\mu} \perp Q$ and $J_2^{\mu} \sim (p_2 - p_3)^{\mu} \perp Q$ and finally pseudovector $J_5^{\mu} \sim \epsilon(\mu, p_1, p_2, p_3)$. Dependence on hadronic currents remain in calculation of polarimetric vectors.

• Model independent methods, template methods, neural networks, multidimensional signatures. It was easier for Cleo. There, $\tau$’s were produced nearly at rest, $\nu_\tau$ four-momentum was easy to reconstruct.

• Fitting in complex situation is ... well complex! Instead of acoplanarity angle in $a_1 - a_1$ case we have 16 such angles.
1. In case of $\tau \rightarrow \rho \nu$ there was one decay plane to define and sign of CP sensitive sinusoid was dependent on sign of $y_+y_-$.  

2. In case of $\tau \rightarrow a_1 \nu$ four planes can be defined. Two for $a_1 \rightarrow \pi \rho^0$ and another two for $\rho^0 \rightarrow \pi^+ \pi^-$ decays.  

3. We end up with 4 (or 16) angular distributions at number of $y_i$ like variables.  

4. That means many sub categories to define sample ...  

5. All distributions are correlated.  

Acoplanarity angles of oriented half decay planes: $\varphi_{\rho^0\rho^0}^*$ (left), $\varphi_{a_1\rho^0}^*$ (middle) and $\varphi_{a_1a_1}^*$ (right), for events grouped by the sign of $y_{\rho^0}, y_{a_1} y_{\rho^0}$ and $y_{a_1} y_{a_1}$ respectively. Three CP mixing angles $\phi_{CP} = 0.0$ (scalar), 0.2 and 0.4. Note scale, effect on individual plot is so much smaller now. But up to 16 plots like that have to be measured, correlations understood. But physics model depends on 1 parameter only and effect of $\phi_{CP}$, the Higgs mixing scalar pseudoscalar angle, is always a linear shift.
Similar/supplementary projects.

Results relevant for fitting and for $\tau$ leptons.


2. Potential for optimizing Higgs boson CP measurement in $H$ to tau tau decay at LHC and ML techniques, R. Jozefowicz, E. Richter-Was and Z. Was, arXiv:1608.02609


• Biases in art, Giuseppe Arcimboldo (1572 - 1593).

• Result depend on model assumptions. Models inspired with results ...
  Fitting setup $\rightarrow$ biases.

• Our algorithms are far less elaborate than human eye/brain.

• That may look worrisome.
We are not alone with the problem

Figure 2: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don’t.

From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

Pattern recognition is an active field and deep concern and not only for us.

Z. Was

Hayama, October, 2016
Summary

- I have presented scattered results where use of computer algebraic methods or pattern recognition techniques (Machine Learning) was necessary.

- My experience with such approaches started in 1996 in MinamiTateya group.

- Working on my talk was inspiring to myself. Also, it was not easy to select slides for a coherent presentation.

- In fact, I am not sure if I was able to send the message: computer algebra methods → correct huge expressions → loss of control on what should come out → how to understand/interpret/use → we are not alone with such difficulties → how to avoid detection of non-existent...

- Manpower/training is an essential issue for continuity of projects.

- The challenges are more for newcomers, who may have missed long years of rewarding failures.

Z. Was

Hayama, October, 2016