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Brief Manual

of

GRACE System

Version 2.0β

(for tentative use)

MINAMI-TATEYA collaboration

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Contents

1 Introduction

In order to calculate the cross section of the elementary processes automatically GRACE system was developed in 1987 and is being implemented gradually. GRACE version 1.0[1] was released at the beginning of 1993, whose graph generation part was coded in PASCAL. At the beginning of 1995 a new graph drawer `gracefig` was developed and installed into GRACE version 1.1. For the sake of portability to any computer systems the graph generation part was coded again in C and improved so as to be able to generate the multi-loop diagrams[2]. GRACE version 2.0 [1] including this new graph generation program `grc` was tentatively released in November 1995.

This manual will show only how to use GRACE version 2.0, whose details can be found in GRACE manual[1] since the basic features of version 2.0 is quite similar to that of version 1.0. The program package BASES/SPRING version 5.1 is used for the numerical integration and event generation, whose detailed description can be found in reference 6.

An interactive version GRACE++ is also available, which are supported in terms of the command interpreter `KUIP`[3] developed at CERN and provides an identical environment to PAW^{++}[4] to users.

In the next section a concept of GRACE is described and the usage of GRACE will be shown in the section 3.

2 A Concept of GRACE

GRACE system consists of the following different four subsystems:

- **Graph generation**
  All Feynman graphs are generated for an elementary process, specified by the names of particles in the initial and final states and the order of perturbation.
  - **Drawing graphs**
    The program **gracefig** draws all generated Feynman graphs on the X-terminal in terms of `OSF/Motif` and allows us to select some graphs and also to make a EPS file to print them on a piece of paper.
- **FORTRAN code generation**
  The numerical value of the differential cross section is calculated by the helicity amplitude formalism[5], whose FORTRAN source code is generated in

[1] GRACE version 2.0 was developed by the following members:

System construction:	T.Ishikawa, S.Kawabata	Graph generation:	T.Kaneko
Graph drawer:	S.Kawabata	Code generation:	T.Kaneko, T.Ishikawa
Integration:	S.Kawabata	Event generation	S.Kawabata
Kinematics Lib.:	Y.Kurihara, Y.Shimizu	CHANEL Lib.:	H.Tanaka
Electroweak part:	T.Kaneko	QCD part:	K.Kato

a suited form for the numerical integration and event generation program `BASES/SPRING v5.1`[6]. In addition to this, a Makefile is also generated for the gauge invariance test, the numerical integration, and the event generation.

  - **Gauge invariance test**
    Before calculating the total cross section a minimal test of program source should be done. For this purpose the gauge invariance test is prepared in GRACE system.

- **Numerical integration**
  The numerical integration of the differential cross section over the phase volume is carried out by the program `BASES`. The probability information is automatically produced and saved in the file `bases.data`, according to which the event generation is done. A special care should be taken in this step is to watch the numerical stability of the integral. (see ref. 6)

- **Event generation**
  Once the numerical integration converges with a good numerical stability, we can generate events in terms of the program `SPRING`. The program `SPRING` samples a hypercubes in the phase volume of the integral variables according to the probalility information made in the integration step, and sample and test a point in the hypercube. Each time when the program `SPRING` accepts a point in the phase volume as an event, the program control returns to the main program `mainsp`, where the four vectors of final particles are to be calculated and saved in a file. For this purpose users must modify the main program `mainsp.f` before the compilation. (see ref. 6)

GRACE system is normally used by following the order of above items, which are described in the next section.

3 How to use GRACE

Before using GRACE, we have to set path to the directory where GRACE system is installed.
For example

```
set path = ( $path /user/local/grace )
```

It is recommended to create a new directory for the calculation of one physical process. For the process $e^+e^- \to W^+W^-\gamma$, we create a directory `eewwa` and move to the directory as below:

```
grace% mkdir eewwa
grace% cd eewwa
```

3.1 Definition of physical process

Before starting the graph generation it is necessary to define the physical process, which is given in the file name "`in.proc`". The content of the file `in.proc` for the process $e^+e^- \to W^+W^-\gamma$ is as follows;

```
Model="non_eH.mdl";
Process;
ELWK=3;
QCD=0;
  Initial={electron,positron};
  Final  ={photon,W-plus,W-minus};
Kinem="3002";
Pend;
```

3.1.1 Model file

In the first line the model file is defined. Two model files are prepared, one is `all.mdl` for the full standard model and another is `non_eH.mdl` for a specific model where the coupling between electron and Higgs is suppressed. Since the electron-Higgs coupling is so small that its contribution to the total cross section is usually negligible, the specific model `non_eH.mdl` is recommended to use normally because of an economical use of the computer resources. When, however, the result with a high precision is required, the full standard model should be taken. See also subsection 3.5.2.

3.1.2 Order of perturbation

The order of perturbation are to be defined; `ELWK` for the electroweek order and `QCD` for the QCD order.

3.1.3 Initial and final states

In the lines for the initial and final states, the particle names are to be given for each state. The ordering of the particles in each state is very important since the built-in-kinematics assigns the particle numbers according to the order appeared. When, for example, the interesting process is a 2-body to 3-body reaction, the kinematics defines the particle numbers as follows:

$$p_1 + p_2 \to p_3 + p_4 + p_5$$

In our example of $e^+e^- \to \gamma W^+W^-$ the input file `in.proc` defines the particle numbers like as e^- (p_1), e^+ (p_2), γ (p_3), W^+ (p_4) and W^- (p_5). This ordering of particles is very scensitive to the convergency of integration especially when there are massless particles in the final state.

GRACE system uses a set of particle names as given below:

Bosons	name	description
W^+	`W-plus`	Weak boson with charge +1
W^-	`W-minus`	Weak boson with charge -1
Z^0	`Z`	Neutral weak boson
γ	`photon`	Photon
g	`gluon`	Gluon

Fermions	name	description
e^-	`electron`	Electron
e^+	`positron`	Positron
μ^-	`muon`	Muon
μ^+	`anti-muon`	Anti-muon
τ^-	`tau`	Tau lepton
τ^+	`anti-tau`	Anti-tau lepton
ν_e	`nu-e`	Electron neutrino
$\bar{\nu}_e$	`nu-e-bar`	Anti-electron neutrino
ν_μ	`nu-mu`	Muon Neutrino
$\bar{\nu}_\mu$	`nu-mu-bar`	Anti-muon neutrino
ν_τ	`nu-tau`	Tau neutrino
$\bar{\nu}_\tau$	`nu-tau-bar`	Anti-tau neutrino

Quarks	name	description
u	`u`	Up quark
$\bar{u}$	`u-bar`	Anti-up quark
d	`d`	Down quark
$\bar{d}$	`d-bar`	Anti-down quark
s	`s`	Strange quark
$\bar{s}$	`s-bar`	Anti-strange quark
c	`c`	Charm quark
$\bar{c}$	`c-bar`	Anti-charm quark
b	`b`	Bottom quark
$\bar{b}$	`b-bar`	Anti-bottom quark
t	`t`	Top quark
$\bar{t}$	`t-bar`	Anti-top quark

Scalars	name	description
ϕ	`Higgs`	Higgs scalar
χ^+	`chi-plus`	Charged Goldstone boson with charge +1
χ^-	`chi-minus`	Charged Goldstone boson with charge -1
χ^3	`chi-3`	Neutral Goldstone boson

Ghosts	name	description
c^+	`C-plus`	Faddeev-Popov ghost associated with W^-
$\bar{c}^-$	`C-plus-bar`	Anti-Faddeev-Popov ghost associated with W^-
c^-	`C-minus`	Faddeev-Popov ghost associated with W^+
$\bar{c}^+$	`C-minus-bar`	Anti-Faddeev-Popov ghost associated with W^+
c^Z	`C-Z`	Faddeev-Popov ghost associated with Z^0
$\bar{c}^Z$	`C-Z-bar`	Anti-Faddeev-Popov ghost associated with Z^0
c^γ	`C-A`	Faddeev-Popov ghost associated with photon
$\bar{c}^\gamma$	`C-A-bar`	Anti-Faddeev-Popov ghost associated with photon
c^g	`C-g`	Faddeev-Popov ghost associated with gluon
$\bar{c}^g$	`C-g-bar`	Anti-Faddeev-Popov ghost associated with gluon

3.1.4 Kinematics code number

The code number of kinematics routine in the kinematics library should be given, which are described in Appendix. As mentioned in subsection 3.1.3, the order of particles in the initial and final states plays a sensitive role for a good convergency of the numerical integration. For defining the order of particles, see Appendix carefully.

3.2 Graph generation

The graph generation procedure starts by typing the command `grc`:

```
grace% grc
```

All Feynman graphs for the process are generated and their information is saved in the file `out.grf`.

3.3 Drawing graphs

Although this step can be skipped if users do not want to see graphs, it is recommended to do this step to confirm the input. One can also print Feynman graphs at a postscript printer. A Feynman graph drawer `gracefig` is initiated by the command:

```
grace% gracefig
```

The procedure `gracefig` reads the graph information from the file `out.grf` and analyzes the structure of each graph, and then creates two windows, the control window and the drawing window for the graphs.

3.3.1 Drawing window of `gracefig`

On the drawing window the first $m \times n$ graphs, appeared in the unitary gauge, are drawn, where the number of lines m and the number of columns n are determined automatically by the size of screen and the total number of graphs. When the size of the drawing window is changed, the number of columns n is conserved while the number of lines m is adjusted so as to fit to the height of the window.

3.3.2 Control window of `gracefig`

In the control window several buttons and pull down menus are displayed for the functions listed the table.

3.3.3 To draw some selected graphs

There are two modes, "*Drawing Mode*" and "*Select Mode*". When `gracefig` starts, all Feynman graphs appeared in unitary gauge are displayed in the *drawing mode*. If one wants to see or print only some typical graphs, for instance, one can select these graphs in the *select mode* by clicking their pictures in the drawing window. The backgrounds of the selected graphs are darken in the *select mode*. After selecting graphs, only the selected graphs can be displayed on the drawing window in the *drawing mode* by selecting "*Selected graphs*" in the graph menu.

Button / Sub-menu	Function
Quit	Exit from `gracefig` .
Next page	Show next page.
Previous page	Show last page.
+5 page	Show 5-th page next.
-5 page	Show 5-th page previous.
Scale up	Make the size of graph larger. The number of graphs in a line: $n \to n-1$.
Scale down	Make the size of graph smaller. The number of graphs in a line: $n \to n+1$.
Option menu	Pull down menu.
/ *Graph number*	On/Off display of graph number.
/ *Particle number*	On/Off display of particle number.
/ *Vertex number*	On/Off display of vertex number.
/ *Particle name*	On/Off display of particle name.
/ *Line number*	On/Off display of internal line number.
Graph menu	Pull down menu.
/ *Covariant Gauge*	Show graphs in covariant gauge.
/ *Unitary Gauge*	Show graphs in unitary gauge (Default).
/ *Selected Graphs*	Show only the selected graphs.
Mode Selection	Pull down menu.
/ *Drawing Mode*	Default mode to draw the graphs.
/ *Select Mode*	To select a graph, click on a graph you choose.
EPS output	Output the current page on a file `grcfig*.eps` in eps format.

3.3.4 EPS output and insertion of picture in a text

When "EPS output" button is pressed, the EPS file for the current page in the drawing window is created. The name of EPS file is "grcfig*nn*.eps", where *nn* is the sequential number starting from 0, and one EPS file corresponds to one page of graphs. At the top of the EPS file `grcfig`*nn*`.eps` the information about the picture size is written:

```
%%BoundingBox: 0 0 600 599
%% left_margin(    30.0) width (     600) right_margin (    30.0)

 0.91  0.91 scale
```

The size of print paper is assumed to be 600 pixel in width (A4). The numbers for the left and right margins and width are given in unit of pixel on the drawing window. In order to fit the picture on the window to A4 paper, the scale factor is necessary, which is equal to the ratio (0.91) of the width (600) of A4 paper to

the sum (660) of the sizes of left_margin, width and right_margin on the drawing window.

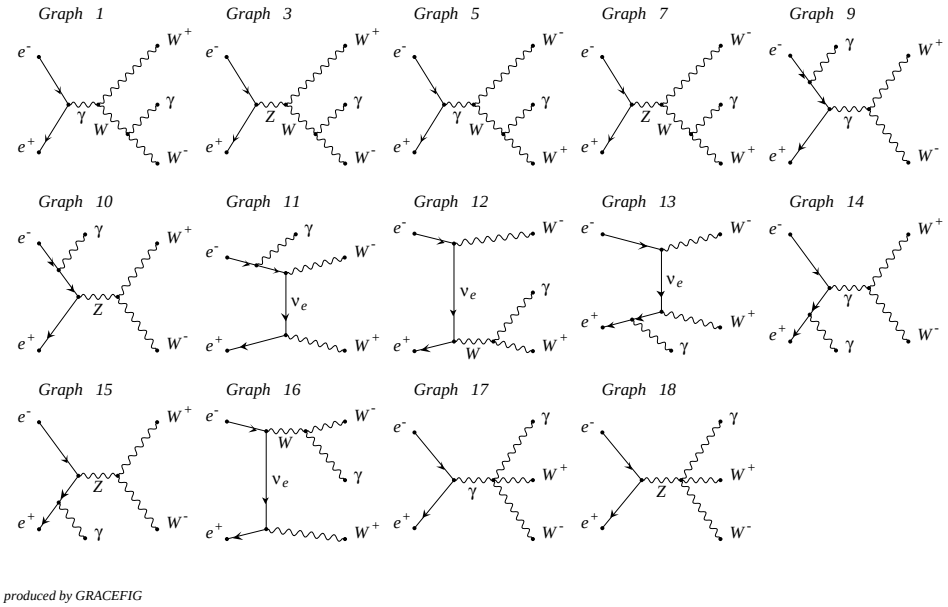

Fig.1 An example of drawn graphs for $e^+e^- \to \gamma W^+W^-$.

In order to insert this picture in a text in terms of LaTex, the size of BoundingBox and scale factor should be calculated. If one wants to insert this picture with the size of 70% of A4 picture, the scale factor and the size of BoundingBox should be changed as ($0.91 \times 0.7 = 0.64$) and ($600 \times 0.7 = 420$, $399 \times 0.7 = 280$), respectively. Then the command in the LaTex file

```
\centerline{\epsfig{file=grcfig0.eps}}
```

produces the 70% size of picture in the text as shown in Fig.1.

3.4 Source code generation

After the graph generation, FORTRAN code and Makefile are generated for the gauge invariance check, the numerical integration and the event generation by typing the command:

```
grace% grcfort
```

3.4.1 Generated source code

There are three kinds of program components. The first is for the amplitude calculation, the second is for the integration by BASES and the third is for the event generation by SPRING.

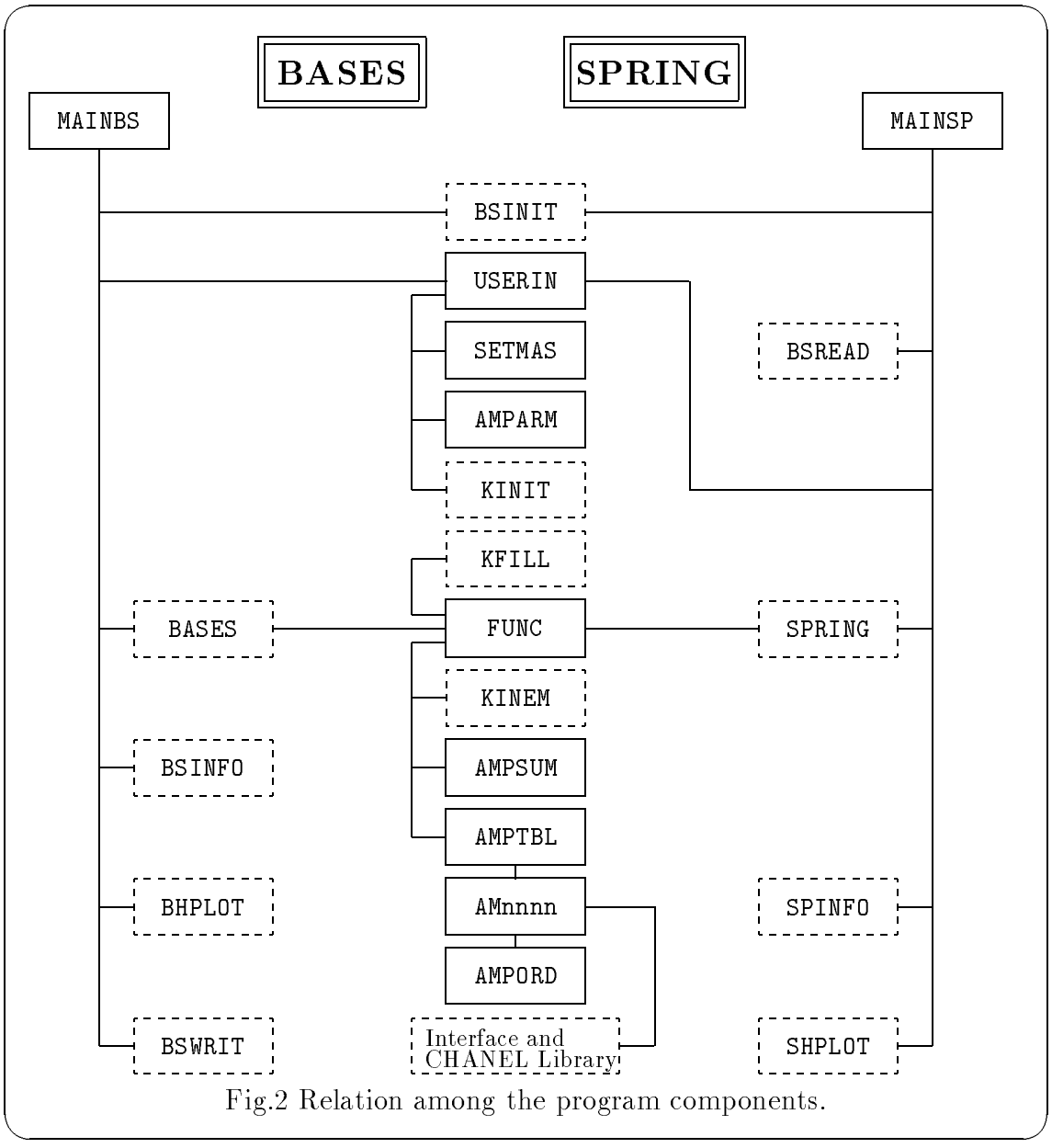

Fig.2 Relation among the program components.

The interrelation among the subprograms generated by GRACE is depicted in Fig.2, where those subprograms in the `solid box` are automatically generated by GRACE, while those in the `dashed box` are already contained in other program packages BASES/SPRING, kinematics library, interface program library to CHANEL, and program package CHANEL. This figure omits a few minor components. The components located

in the middle of the figure are commonly used in BASES and SPRING. The program specifications of the libraries BASES/SPRING, the interface to CHANEL and program package CHANEL are described in references 1 and 6. A brief descriptions of generated subprograms are given below:

(1) Main programs.

MAINBS	*(main)*	is the main program for the integration.
MAINSP	*(main)*	is the main program for the event generation.
GAUGE	*(main)*	is the test main program for the check of gauge invariance.

(2) A set of program components for the integration by BASES and for the event generation by SPRING.

USERIN	*(subroutine)*	controls initialization.
SETMAS	*(subroutine)*	defines masses and decay widths of particles.
AMPARM	*(subroutine)*	defines coupling constants and others.
KINIT	*(subroutine)*	initializes integration parameters, histograms, kinematics and user's parameters.
FUNC	*(function)*	calculates the numerical values of differential cross section.
KINEM	*(subroutine)*	derives particle four momenta from the integration variables.
KFILL	*(subroutine)*	fills values in histograms and scatter plots.
AMPTBL	*(subroutine)*	calls AMnnnn to calculate amplitudes.
AMPSUM	*(subroutine)*	sums matrix elements over the helicity states. A matrix element is the square of the sum of amplitudes.
AMnnnn	*(subroutine)*	calculates amplitude of the nnnn-th graph, where the number nnnn of the routine name is equal to the graph number.
AMPORD	*(subroutine)*	arranges amplitudes.
incl1.f	*(include file)*	defines the common variables for masses, amplitude tables *etc.*
incl2.f	*(include file)*	defines the work space for AMPTBL.
inclk.f	*(include file)*	transfers values of masses and a few constants.
USROUT	*(subroutine)*	prints the amplitude summary table. Only used by gauge.

Although the program components KINIT, KFILL, FUNC and KINEM are created automatically by GRACE, they need modifications. Especially when the built-in kinematics is not suited for the problem at hand, the kinematics routines KINIT and KINEM must be written by the user.

The subprograms USERIN and FUNC are used both in the numerical integration by BASES and the event generation by SPRING. In the routine USERIN, subroutines

KINIT is called to initialize kinematics and other parameters. The routines SETMAS and AMPARM are called for the initialization of physical parameters and amplitude calculation.

The function subprogram FUNC is used for calculating the numerical value of differential cross section, where subroutine KINEM is called for calculating four momentum vectors of external particles and subroutines AMPTBL and AMPSUM are called for the amplitude calculation. The detailed specifications for these program components are described in references 1 and 6.

3.4.2 Edit FORTRAN source code

We are improving the system so that we may have as less chance to edit files directly as possible, but several parts are still remained to be modified. Also, when the user wants to make histograms, one must edit source code to call XHINIT, DHINIT, XHFILL, DHFILL. Here, *HINIT's are to be called for initialization of histogram, and *HFILL fills a value on the histogram. The first character X and D specify the 1–dimension histogram and the 2-dimensional scatter plot, respectively.

(1) **Initialization routine** KINIT

One can choose options and parameters by editing KINIT as follows:

(a) Modification of physical parameters
Some physical parameters are to be modified according to the user's will.

(i) Parameters for kinematics
- Center of mass energy in GeV
- Various kinematical cuts etc.

(ii) Option flags in built-in kinematics.
The variety of these flags depends on the nature of kinematics at hand. Some examples are as follows:
- If an integration variable corresponds to an invariant mass, M^2, then one may choose to use either M^2 or $\log M^2$ in the kinematics by setting a flag in KINIT.
- The flag to suppress the integration over rotation around beam axis.

The list of such parameters are given in Appendix.

(b) Integration parameters which control the action of BASES can be altered in KINIT. Among them the following parameters are usually not to be altered:

NDIM	The number of dimensions of integral
NWILD	The number of wild variables
XL(i),XU(i)	The lower and upper bounds of integration variable X(i)
IG(i)	The grid optimization flag for i-th variable

The following parameters are to be defined according to user's will:

NCALL	The number of sampling points per iteration
ACC1	The expected accuracy for the grid optimization step
ITMX1	The maximum iteration number for the grid optimization
ACC2	The expected accuracy for the integration step
ITMX2	The maximum iteration number for the integration step

(c) Initialization of histograms and scatter plots

If the user needs histograms for some physical variables, the initialization routines XHINIT and DHINIT should be called for each histogram and scatter plot, respectively.

The system automatically generates source code to call these so as to make histograms with respect to each integration variables, X(I), I=1 $\cdots$ NDIM and the energies and angles of final particles. The meaning of each X(I) is given in the corresponding section in the Appendix, where section name is the code number of kinematics.

If the user does not need these default histograms, one deletes the corresponding lines in KINIT. When the user wants to make histograms for some distributions, one appends lines to call XHINIT or DHINIT. These routines are to be called as:

```
CALL XHINIT( ID#,
.            lower_limit, upper_limit, # of bins,
.            'Title of histogram' )
```

and

```
CALL DHINIT( ID#,
.            x-lower_limit, x-upper_limit, # of x bins,
.            y-lower_limit, y-upper_limit, # of y bins,
.            'Title of scatter plot' )
```

(2) **Filling histograms and scatter plots in KFILL**

In the generated KFILL by GRACE, histograms for all integration variables and scatter plots for all combinations of them are to be filled by calling XHFILL and DHFILL, respectively. If one has changed the initialization of histograms and scatter plots in KINIT, their filling parts should be also changed. These routines are to be called as:

```
CALL XHFILL( ID#, V, FUNC )
```

and

```
CALL DHFILL( ID#, Vx, Vy, FUNC )
```

(3) **Function subprogram** `FUNC`

If colored particles are included in the process, one must include the strong coupling constant as a multiplicative factor in the function. The generated function properly includes the color factors while it assumes $g_s = 1$. This is because the argument of $\alpha_s(\mu^2)$ cannot be determined uniquely in general. There is a line in the subroutine `KINEM` to define a variable `YACOB`. One can multiply g_s^n as

$$\texttt{YACOB} = \texttt{YACOB} * \cdots$$

below that line.

(4) **Subprogram** `SETMAS`.

(a) The physical parameters like masses and widths of particles are defined in `SETMAS` subroutine. If one wants to change the value of m_{top}, m_{Higgs} and so on, the user should edit this file.

(b) Selection of graphs.

The contribution to the total cross section from a set of some graphs can be calculated by setting only their flags `JSELG = 1` defined in `SETMAS`. It should be noted that this facility be independent from the graph selection in the Feynman graph drawer `gracefig`.

(5) **Output file for generating events.**

In the event genration the four momentum vectors of final particles are usually saved in a file. The user should save them according to his data structure of the event. In the main program `MAINSP` the subprogram `SPRING` is called for one event generation, after which one can save the event information. See also section 3.7.

The items (1c), (2) and (5) will be improved in near future, while the other items are unavoidable.

3.5 Before calculating the cross section $\cdots$

3.5.1 Make

As described in previous subsection, a Makefile is also generated by the command `grcfort`. Befor running any procedures, they should be compiled and linked by the command `make`:

```
grace% make
```

3.5.2 Test of the gauge invariance

Before calculating the cross section a minimal test of program source should be done. In GRACE system the gauge invariance test is prepared for that purpose, where one point in the phase space is sampled and the numerical values of the differential cross section at the point are calculated both in the covariant and unitary gauges, and then the difference between these two values is tested. The executable `gauge` is already created and is executed by the following commands:

```
grace% gauge
```

This reports the contribution of each graph to the cross section as well as the values of differential cross section at the point in both gauges.

```
 . . . . .
ans1   =  .4688388270357785
 . . . . .
       ..............
ans2   =  .468838827035779
 . . . . .
       ..............
ans1/ans2 - 1 =  -9.992007221626408E-16
 . . . . .
```

The numerical values `ans1` and `ans2` are of the differential cross section in the unitary and covariant gauges, respectively. If the numerical values of differential cross section are identical, it passes this check.

The relative difference should be around 10^{-16} in the full standard model because the calculation is done with double precision. If, however, the `non_eH.mdl` is taken, this difference is not enough accurate (around 10^{-12}) due to suppressed coupling between the electron and Higgs.

It should be noted that this test does not guarantee a complete gauge invariance even though it could give consistency between the two gauges, since it tests only at a specific point in the phase space. It is recommended to test the gauge invariance at several points in the phase space.

3.6 Integration

After examination of the subprograms `KINIT` and `KFILL` and a successful test of gauge invariance, we can proceed to the numerical integration by `BASES`, whose details can be found in reference 6. By `make`, we have already made the executables `integ` for the integration and `spring` for event generation.

For integration the command `integ` is used.

```
grace% integ
```

The integration package `BASES` prints the integration parameters and the result on the screen at every end of the iteration, and also it writes the same lines together with the histogram output on the file named `bases.result` at the end. If one uses built-in kinematics, the cross section is given in unit of *pb*.

The numerical integration is performed by two steps, one is the grid optimization step and another the integration step. In the former step the grids are optimized for the integrand, while in the latter step an accurate estimate of integral is calculated with frozen grids determined in the former step.

Before termination of the integration procedure, `BASES` writes the probability information on a binary file `bases.data`, which is used for the event generation.

During the integration, the system may issue `WARNING` messages, if the convergence is not well established. However, this diagnostics message is not absolute, so that the user must be careful for the check of the integration.

It is recommended to look at the integration result carefully, especially over the convergency behaviors both for the grid optimization and integration steps. When the accuracy of each iteration fluctuates, iteration by iteration, and, in some case, it jumps up suddenly to a large value compared to the other iterations, the resultant estimate of integral may not be reliable. There are two possible origins of this behavior; one is due to too small number of sampling points and the other due to an unsuitable choice of the integration variables for the integrand.

3.7 Event generation

Since the executable `spring` is created by the make command already, the event generation starts by typing

```
grace% spring
```

Then `SPRING` reads the probability information from the binary file `bases.data` and asks the number of events with the following prompt:

```
Number of events ?
```

Here, the user must type the number of events to be generated.

3.7.1 Algorithm of event generation

The event generation program `SPRING` samples a hypercube according to its probability calculated in the integration step and test if this point is accepted by comparing the probability at the point to the maximum probability in the hypercube. Since there is some possibility that the generation loop may get into an infinite loop, it is terminated by the maximum number of trials. We call this "failed generation". The event generation will run until a given number of events are generated or the number of failed generations exceeds its given maximum `mxtry`, whose default value is 50.

In order to estimate the computing time for the event generation, it is recommended to use the expected generation time given in the computing time information of `BASES` output, where the generation efficiency of 70% is assumed.

3.7.2 Analysis of events

To analyze the generated events and to save them on a file, one should open this file at the beginning and should insert these program codes just after calling `spring` in the main program `MAINSP`.

To make histograms of some variables of the generated events, one has to initialize them just before the generation loop and fill them after each call of `spring`. This kind of histograms is called as an "*additional histogram*" in contrast with the "*original histogram*" which is filled both in the integration and event generation steps. In the original histograms we can see how the generated events reproduce those distributions produced by the integration, while in the additional histogram only the fequency distribution can be seen as in the usual histogram.

```
*     Open the output file for the events
      . . . . . .

*     Initialize the additional histograms by
        call xhinit( id, ... ).

      do 100 nevnt = 1, mxevnt

         call spring( func, mxtry )

*    ------------------------------------------------
*        analyze the event and
*        fill the additional histograms by
*          call xhfill( id, .... ).
*    ------------------------------------------------

         do 90 k = 1 , nextrn
            write(6,*) (vec(j,k),j=1,4)
   90    continue
  100 continue
```

3.7.3 Output from SPRING

The output from SPRING is written on the file named `spring.result`, which consists of the general information, original and additional histograms, scatter plots, and number of trials distribution. In the number of trials distribution we can see the generation efficiency. It is recommended to see if this distribution has a sharp peak at the first bin. If not the case, we should perform the integration again with more sampling points or with more iterations.

A Kinematics

Below is the list of built-in kinematics in the system. This will includes more candidates in the future. The name of section shows the code number of the kinematics.

Normally, if one uses built-in kinematics, the cross section is given in unit of pb.

code number	contents
`2001`	2-body $\to$ 2 body in CM frame No t-channel singularity.
`2002`	2-body $\to$ 2 body in CM frame With t-channel singularity. (forward peak)
`2003`	2-body $\to$ 2 body in CM frame With t-channel singularity. (forward-backward peak)
`3001`	2-body $\to$ 3 body in CM frame , Sequential decay type $1+2 \to 3+(4+5) \to 3+4+5$. Simple phase space.
`3002`	2-body $\to$ 3 body in CM frame , Sequential decay type $1+2 \to 3+(4+5) \to 3+4+5$. Particle-3 is a radiative photon from initial particles.
`3003`	2-body $\to$ 3 body in CM frame , Sequential decay type $1+2 \to 3+(4+5) \to 3+4+5$. Particle 4 and 5 make a resonance.
`3004`	2-body $\to$ 3 body in CM frame , Sequential decay type $1+2 \to 3+(4+5) \to 3+4+5$. Invariant mass of 4 and 5 behaves $\sim 1/M^2$.
`3005`	2-body $\to$ 3 body in CM frame , Particle 5 is produced at central by 'fusion'. $1 \to 3+A$, $2 \to 4+B$, $A+B \to 5$.
`3006`	2-body $\to$ 3 body in CM frame , Radiative processes $1+2 \to 3(\gamma)+4+5$, both initial and final radiation can be treated.
`3007`	2-body $\to$ 3 body in CM frame , Double-radiative processes $1+2 \to 3(\gamma)+4(\gamma)+5$
`3008`	2-body $\to$ 3 body in CM frame , Three phton processes $1+2 \to 3(\gamma)+4(\gamma)+5(\gamma)$
`3009`	2-body $\to$ 3 body in CM frame , General purpos kinematics. It can be used for almost all processes except radiative one.

code number	contents
`4001`	2-body $\to$ 4 body in CM frame, a pair of sequential decay type $1+2 \to (3+4)+(5+6) \to 3+4+5+6$ No t-channel singularity.
`4002`	2-body $\to$ 4 body in CM frame, a pair of sequential decay type $1+2 \to (3+4)+(5+6) \to 3+4+5+6$ With t-channel singularity.
`4003`	2-body $\to$ 4 body in CM frame, 'fusion' type $1+2 \to (3+A)+(4+B); A+B \to 5+6$
`4004`	2-body $\to$ 4 body in CM frame, General purpos kinematics.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. Generally, the kinematics assumes

$$p_1 + p_2 \to p_3 + p_4 + \cdots \qquad (2-\text{body scattering}),$$

$$p_1 \to p_2 + p_3 + p_4 + \cdots \qquad (\text{decay of a particle}),$$

and so on. Here the assignment of particles keeps the order of particles in the input data at the stage of graph generation.

The Lorentz invariant phase space for final n-body ($A \to 1+2+3+\cdots+n$) is defined by

$$d\tilde{\Gamma}_n = (2\pi)^4 \delta^{(4)}\left(\sum_{in} p - \sum_{out}^{n} p\right) \prod_{out}^{n} \frac{d^3 p_j}{(2\pi)^3 2E_j} \equiv \frac{1}{(2\pi)^{3n-4}} d\Gamma_n$$

$$d\Gamma_n = \delta^{(4)}\left(\sum_{in} p - \sum_{out}^{n} p\right) \prod_{out}^{n} \frac{d^3 p_j}{2E_j}.$$

The chain relation for the phase space is useful($0 < k < n-1$):

$$d\Gamma_n(A \to 1{+}2{+}3{+}\cdots) = d\Gamma_{k+1}(A \to 1{+}\cdots{+}k{+}q) \frac{dQ^2}{2\pi} d\Gamma_{n-k}(q \to (k{+}1){+}(k{+}2){+}\cdots)$$

where $q^2 = Q^2$.

When p_b and p_c are in the center-of-mass system and $p_a = p_b + p_c$,i.e., $\vec{p}_a = \vec{0}$, we use the following notations

$$d\Gamma_2 = d\Gamma_{\rm CM}(a; bc) = \delta^{(4)}(p_a - p_b - p_c) \frac{d^3 p_b}{2E_b} \frac{d^3 p_c}{2E_c}$$

and write it by angular variables:

$$d\Gamma_{\rm CM}(a; bc) = \frac{\beta(a; bc)}{8} d\Omega_{\rm CM}(a; bc) = \frac{\beta(a; bc)}{8} d\cos\theta_{b,(bc)} d\phi_{b,(bc)}$$

Here

$$\begin{aligned}\beta(a;bc) &= \frac{2P}{E_a} \\ &= \frac{1}{E_a}\sqrt{(E_a+m_b+m_c)(E_a-m_b-m_c)(E_a+m_b-m_c)(E_a-m_b+m_c)}\end{aligned}$$

and the suffix (bc) denotes that the angles θ_b, ϕ_b are defined in the center-of-mass system. Angles in the laboratory frame have no suffix.

A.1 2001

Kinematics `2002` and `2003` is variation of this one and the user can also consult them in the following sections.

Description of the kinematics

This is the simple kinematics for 2 to 2 process in the center-of mass system. Integration variables are naturally the polar angle θ and azimuthal angle ϕ with respect to the incoming particles.

If there is no strong peak to some directions, e.g., to the forward direction, this works well. Also user can introduce forward and backward angle cutoff as options.

We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll}\text{Particle-1} & (0,0,+P,E_1) \\ \text{Particle-2} & (0,0,-P,E_2)\end{array}$$

where P is the positive value determined by m_1, m_2, W. The angles θ and ϕ represent the direction of particle-3. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

Phase space is in the center-of-mass system and it is given by

$$d\tilde{\Gamma}_2 = \frac{1}{(2\pi)^2} d\Gamma_{\rm CM}(12;34) = \frac{\beta(12;34)}{8(2\pi)^2} d\cos\theta d\phi.$$

Meaning of `X()` for BASES integration

$$\begin{array}{ll}\texttt{X(1)} & \cos\theta = 2\texttt{X(1)} - 1 \\ \texttt{X(2)} & \phi = 2\pi\texttt{X(2)}\end{array}$$

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section
   - `W` R*8 (200.0) : Center of mass energy.
   - `COSCUT(1)` R*8 (-1.0) : Minimum of $\cos\theta$.
   - `COSCUT(2)` R*8 (+1.0) : Maximum of $\cos\theta$.
2. Physics control section
   - `NDIM` I*4 (2) : If you want to suppress ϕ-integration, replace the definition of `NDIM` by the following:

     ```
     NDIM = 1
     ```

     Then ϕ is fixed to 0.0.
3. `BASES` control section
   - `ITMX1` I*4 (5) : These three values control `BASES` integration. See document of `BASES` for details.
   - `ITMX2` I*4 (5) :
   - `NCALL` I*4 (5000) :
   - `NX` I*4 (50) : Value `NX` control the histograms.

A.2 2002

This is the kinematics similar to `2001` and only the difference is quoted here.

Description of the kinematics

In this kinematics, with flag `ICOST=+1`, the cross section is assumed to have $\sim 1/t$ singularity, where $t = (p1 - p3)^2$. New integration variable is introduced as

$$\begin{aligned} D &= -t + m_1^2, \\ &= 2(E_1 E_3 - P P_3 \cos\theta) - m_3^2. \end{aligned}$$

The variable is transformed into

$$\begin{aligned} dD &= 2PP_3 d\cos\theta, \\ dD/D &= d(\log D). \end{aligned}$$

Then the phase space in `2001` is replaced by

$$\begin{aligned} d\cos\theta &= \frac{D}{2PP_3} d(\log D), \\ &= \frac{D}{2PP_3} \log(D_{max}/D_{min}) d\eta \end{aligned}$$

where $D = D_{min}(D_{max}/D_{min})^{\eta}$ for $0 < \eta < 1$.

Meaning of `X()` for BASES integration

`X(1)` Momentum transfer in t-channel, $D = -(p1 - p3)^2 + m_1^2$.
$D = D_{min}(D_{max}/D_{min})^{X(1)}$

`X(2)` the same as in `2001`.

Options in `kinit.f`

Physics control section

- `ICOST` I*4 (1) : Treatment of $\cos\theta$.

A.3 2003

This is the kinematics similar to `2001` and only the difference is quoted here.

Description of the kinematics

In this kinematics, with flag `ICOST=-1`, the cross section is assumed to have $\sim 1/t$ and $\sim 1/u$ singularity, where $t = (p1 - p3)^2$ and $u = (p1 - p4)^2$. New integration variable , D, is introduced as the same as `2002`, but $\cos\theta$ is symmetrized around 90°.

Meaning of `X()` for BASES integration

`X(1)` Momentum transfer in t-channel, $D = -(p1 - p3)^2 + m_1^2$.
$D = D_{min}(D_{max}/D_{min})^{X(1)}$, D is symmetrized around X(1)=0.5.

`X(2)` the same as in `2001`.

Options in `kinit.f`

Physics control section

- `ICOST` I*4 (1) : Treatment of $\cos\theta$.

A.4 3001

Kinematics `3002`, `3003`, `3004`, and `3009` are variation of this one and the user can also consult them in the following sections.

Description of the kinematics

This is the kinematics for 2 to 3 process in the center-of mass system. Here the final state first splits into particle-3 and the system of particles 4 and 5. After that the latter decays into particle-4 and particle-5.

$$1 + 2 \longrightarrow 3 + q \qquad q \longrightarrow 4 + 5$$

Integration variables are the polar angle θ and azimuthal angle ϕ for the first split, those angles for the second split, and the invariant mass of particles 4 and 5. Angles for the first split are defined with respect to the incoming particles and those for the second split are defined in the center-of-mass system of particles 4 and 5 with respect to the momentum direction of the system.

If there is a mass singularity for the two of particles in the final state, it is recommended to assign the two particles to particles 4 and 5. Also user can introduce cutoff for angles and minimum energies as options.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll} \text{Particle-1} & (0, 0, +P, E_1) \\ \text{Particle-2} & (0, 0, -P, E_2) \end{array}$$

where P is the positive value determined by m_1, m_2, W. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

The angles θ_3 and ϕ_3 represent the direction of particle-3. Invariant mass of 4 and 5, Q^2 is another variable.

$$Q^2 = (p_4 + p_5)^2$$

In the center of mass system of particles 4 and 5, angles $\theta_{4,(45)}$ and $\phi_{4,(45)}$ represent the direction of particle-4. The system of particles 4 and 5 are boosted backward to the momentum direction of particle-3.

Phase space is in the center-of-mass system and it is given by

$$\begin{aligned} d\tilde{\Gamma}_3 &= \frac{1}{2\pi} d\Gamma_2(1 + 2 \to 3 + q) dQ^2 d\Gamma_2(q \to 4 + 5) \\ &= \frac{\beta(12; 3q)\beta(q; 45)}{8^2(2\pi)^5} d\cos\theta_3 d\phi_3 dQ^2 d\cos\theta_{4,(45)} d\phi_{4,(45)}. \end{aligned}$$

Here there is no singular behavior, and `IRESN=0` and `ICOS3=0` are assumed.

Meaning of X() for BASES integration

X(1)	$\cos\theta_{4,(45)} = 2$X(1) $- 1$ Polar angle in the CM system of particles 4 and 5.
X(2)	$\phi_{4,(45)} = 2\pi$X(2) Azimuthal angle in the CM system of particles 4 and 5.
X(3)	$\cos\theta_3 = 2$X(3) $- 1$ Polar angle of particle-3
X(4)	Invariant mass of particles 4 and 5, Q^2. Normal. $Q^2 = Q^2_{min} + (Q^2_{max} - Q^2_{min})$X(4)
X(5)	$\phi_3 = 2\pi$X(5) Azimuthal angle of particle-3

Options in kinit.f

Default value is shown in parenthesis.

1. Physical parameter section

   - W R*8 (200.0) : Center of mass energy.
   - COSCUT(1,1) R*8 (-1.0) : Minimum of $\cos\theta_3$.
   - COSCUT(2,1) R*8 (+1.0) : Maximum of $\cos\theta_3$.
   - COSCUT(1,2) R*8 (-1.0) : Minimum of $\cos\theta_4$. Here, θ_4 is the polar angle of particle-4 with respect to the beam axis.
   - COSCUT(2,2) R*8 (+1.0) : Maximum of $\cos\theta_4$.
   - COSCUT(1,3) R*8 (-1.0) : Minimum of $\cos\theta_5$. Here, θ_5 is the polar angle of particle-5 with respect to the beam axis.
   - COSCUT(2,3) R*8 (+1.0) : Maximum of $\cos\theta_5$.
   - ENGYCT(1,1) R*8 (m_3) : Minimum of E_3.
   - ENGYCT(2,1) R*8 (W) : Maximum of E_3.
   - ENGYCT(1,2) R*8 (m_4) : Minimum of E_4.
   - ENGYCT(2,2) R*8 (W) : Maximum of E_4.
   - ENGYCT(1,3) R*8 (m_5) : Minimum of E_5.
   - ENGYCT(2,3) R*8 (W) : Maximum of E_5.
   - AMASCT(1) R*8 (m_4+m_5) : Minimum of Q. Q is the mass of the system of particles 4 and 5.
   - AMASCT(2) R*8 ($W - m_3$) : Maximum of Q.
   - ARESNS(1) R*8 (0.0) : Mass of resonance, this and the next parameter are meaningful only when IRESN=+1.

- ARESNS(2) R*8 (0.0) : Width of resonance.

2. Physics control section

   - NDIM I*4 (5) : If you want to suppress ϕ_3-integration, replace the definition of NDIM by the following:

     ```
     NDIM = 4
     ```

     Then ϕ_3 is fixed to 0.0.
   - IRESN I*4 (0) : Treatment of Q^2. (Do not change.)
   - ICOS3 I*4 (0) : Treatment of θ_3. (Do not change.)

3. BASES control section

   - ITMX1 I*4 (5) : These three values control BASES integration. See document of BASES for details.
   - ITMX2 I*4 (5) :
   - NCALL I*4 (5000) :
   - NX I*4 (50) : Value NX control the histograms.

Related modules

WTOLAB

A.5 3002

This is the kinematics similar to 3001 and only the difference is quoted here.

Description of the kinematics

Here, the particle-3 favors the beam direction, e.g, the photon radiated from initial particles. So in this case, $m_3 = 0$ and $m_1 = m_2$ is assumed.

When the flag ICOS3=1, the kinematics is changed to the case for the radiation of particle-3 along beam axis. If ICOS3=1, the flag IRESN is neglected. Here, the energy of particle-3, E_3, is used instead of Q^2 using

$$Q^2 = W^2 - 2WE_3 + m_3^2 \quad (m_3 = 0)$$

and E_3 is converted into

$$\frac{dE_3}{E_3} = d(\log E_3)$$

to absorb the $1/E_3$ behavior(soft singularity) which appears in the photon radiation.

The angle θ_3 is changed to absorb the collinear singularity, which appears in the form of

$$\frac{1}{D_1 D_2}, \quad D_1 = 2p_1p_3, \quad D_2 = 2p_2p_3.$$

We introduce a variable

$$\tau = \frac{1 + v\cos\theta_3}{1 - v\cos\theta_3} \quad , \quad v = \sqrt{1 - 4m_1^2/W^2},$$

$$y = \frac{1}{4}\left(2 + \frac{\log\tau}{\log\xi}\right), \quad \xi = \sqrt{\frac{1+v}{1-v}}.$$

Here the correspondence is that $y = (0, 1)$ to $\cos\theta_3 = (-1, 1)$ and

$$D_1 = \frac{4E_1E_3}{1+\tau}, \quad D_2 = D_1\tau.$$

Then the phase space in `3001` is replaced by

$$dQ^2 = 2WE_3\log(E_{3,max}/E_{3,min})dx_4$$

where $E_3 = E_{3,min}(E_{3,max}/E_{3,min})^{x4}$ and

$$d\cos\theta_3 = \log(\xi)\frac{D_1D_2}{E_3^2}\frac{1}{2E_1P_1}dy.$$

Meaning of `X()` for BASES integration

`X(1)`, `X(2)`, `X(5)` are the same in `3001`.

`X(3)` Related to $\cos\theta_3$ through the variable y above.
$y = y_{min} + (y_{max} - y_{min})$`X(3)`

`X(4)` Energy of particle-3.
$E_3 = E_{3,min}(E_{3,max}/E_{3,min})^{\texttt{X(4)}}$

Options in `kinit.f`

Physics control section

- `IRESN` I*4 (0) : Treatment of Q^2. (Do not change.)
- `ICOS3` I*4 (1) : Treatment of θ_3. (Do not change.)

A.6 3003

This is the kinematics similar to `3001` and only the difference is quoted here.

Description of the kinematics

When the flag `IRESN=+1`, the amplitude is assumed to have a resonance behavior as

$$\frac{1}{(Q^2 - M_R^2)^2 + M_R^2\Gamma_R^2}.$$

Then the variable is transformed into

$$\frac{dQ^2}{(Q^2 - M_R^2)^2 + M_R^2\Gamma_R^2} = \frac{dt}{M_R\Gamma_R} \qquad \left(t = \arctan\frac{Q^2 - M_R^2}{M_R\Gamma_R}\right)$$

where M_R and Γ_R are the mass and width of the resonance.

Then the phase space in `3001` is replaced by

$$dQ^2 = \frac{(Q^2 - M_R^2)^2 + M_R^2\Gamma_R^2}{M_R\Gamma_R}dt.$$

Meaning of `X()` for BASES integration

`X(1)`, `X(2)`, `X(3)`, `X(5)` are the same in `3001`.

`X(4)` Invariant mass of particles 4 and 5, Q^2.
Resonance. $t = \arctan((Q^2 - M_R^2)/(M_R\Gamma_R))$
$t = t_{min} + (t_{max} - t_{min})$`X(4)`
$Q^2 = M_R^2 + M_R\Gamma_R \tan t$

Options in `kinit.f`

Physical parameter section

- `ARESNS(1)` R*8 (M_W) : Mass of resonance, M_R.
- `ARESNS(2)` R*8 (Γ_W) : Width of resonance, Γ_R.

Physics control section

- `IRESN` I*4 (1) : Treatment of Q^2. (Do not change.)
- `ICOS3` I*4 (0) : Treatment of θ_3. (Do not change.)

A.7 3004

This is the kinematics similar to `3001` and only the difference is quoted here.

Description of the kinematics

When the flag `IRESN=-1`, the amplitude is assumed to contain a pole as $1/Q^2$, and the variable is transformed into

$$\frac{dQ^2}{Q^2} = d(\log Q^2).$$

Then the phase space in `3001` is replaced by

$$dQ^2 = Q^2 d(\log Q^2) = Q^2 \log(Q^2_{max}/Q^2_{min}) dt$$

where $Q^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^t$ for $0 < t < 1$.

Meaning of `X()` for BASES integration

`X(1)`, `X(2)`, `X(3)`, `X(5)` are the same in `3001`.

`X(4)` Invariant mass of particles 4 and 5, Q^2.
$1/Q^2$ behavior assumed.
$Q^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^{\texttt{X(4)}}$

Options in `kinit.f`

Physics control section

- `IRESN` I*4 (-1) : Treatment of Q^2. (Do not change.)
- `ICOS3` I*4 (0) : Treatment of θ_3. (Do not change.)

A.8 3005

Description of the kinematics

This is the kinematics for 2 to 3 process in the center-of mass system. Here a particle 3 emmit particle A and a particle 4 emmit particle B. After that particles A and B collide into particles 5;

$$1 \longrightarrow 3 + A \quad , \quad 2 \longrightarrow 4 + B \quad , \quad A + B \longrightarrow 5$$

Integration variables are the polar angle θ and energy of particles 3 and 4, and azimuthal angle ϕ of particles 3. Angles of 3 and 4 are defined with respect to the incoming particles.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll} \text{Particle-1} & (0,0,+P,E_1) \\ \text{Particle-2} & (0,0,-P,E_2) \end{array}$$

where P is the positive value determined by m_1, m_2, W. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

New integration variable is intruduced as

$$\begin{aligned} Q^2 &= -(p_1 - p_3)^2 \\ &= 2p_1p_3 - m_1^2 - m_3^2 \\ q_0 &= E_1 - E_3 \end{aligned}$$

Phase space is in the center-of-mass system and it is given by

$$d\tilde{\Gamma}_3 = \frac{1}{8(2\pi)^5} \frac{dE_3 d\cos\theta_3 dE_4 d\cos\theta_4 d\phi_3}{\sin\theta_3 \sin\theta_4 |\sin\phi_4|}$$

In this kinematics, with `ICOS3=1` the cross section is assumed to have $\sim 1/Q^2$ singularity. The variable is transformed into

$$\begin{aligned} dQ^2 &= 2PP_3 d\cos\theta_3, \quad (\text{for } \texttt{ICOS3=0}) \\ dQ^2/Q^2 &= d(\log Q^2). \quad (\text{for } \texttt{ICOS3=1}) \end{aligned}$$

Then phase space is replaced by

$$\begin{aligned} d\cos\theta_3 &= \tfrac{1}{2PP_3} dQ^2, \quad (\text{for } \texttt{ICOS3=0}) \\ &= \tfrac{Q^2}{2PP_3} \log(Q^2_{max}/Q^2_{min}) d\eta \quad (\text{for } \texttt{ICOS3=1}) \end{aligned}$$

where $Q^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^\eta$ for $0 < \eta < 1$.

Meaning of `X()` for BASES integration

`X(1)` Energy of $q = p_1 - p_3$

`X(2)` $Q^2 = -q^2$
$Q^2 = Q^2_{min} + (Q^2_{max} - Q^2_{min}) X(2)$ for ICOS3=0
$Q^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^{X(2)}$ for ICOS3=1

`X(3)` $\cos\theta_4 = 2$`X(3)` $- 1$
Polar angle of particle-4

`X(4)` Energy of particle-4

`X(5)` $\phi_3 = 2\pi$`X(5)`
Azimuthal angle of particle-3

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section

   - `W` R*8 (200.0) : Center of mass energy.
   - `COSCUT(1,1)` R*8 (-1.0) : Minimum of $\cos\theta_3$.
   - `COSCUT(2,1)` R*8 (+1.0) : Maximum of $\cos\theta_3$.
   - `COSCUT(1,2)` R*8 (-1.0) : Minimum of $\cos\theta_4$. Here, θ_4 is the polar angle of particle-4 with respect to the beam axis.
   - `COSCUT(2,2)` R*8 (+1.0) : Maximum of $\cos\theta_4$.
   - `COSCUT(1,3)` R*8 (-1.0) : Minimum of $\cos\theta_5$. Here, θ_5 is the polar angle of particle-5 with respect to the beam axis.
   - `COSCUT(2,3)` R*8 (+1.0) : Maximum of $\cos\theta_5$.
   - `ENGYCT(1,1)` R*8 (m_3) : Minimum of E_3.
   - `ENGYCT(2,1)` R*8 ($\frac{s+m_3-(m_4+m_5)^2}{2W}$) : Maximum of E_3.
   - `ENGYCT(1,2)` R*8 (m_4) : Minimum of E_4.
   - `ENGYCT(2,2)` R*8 ($\frac{s+m_4-(m_5+m_3)^2}{2W}$) : Maximum of E_4.
   - `ENGYCT(1,3)` R*8 (m_5) : Minimum of E_5.
   - `ENGYCT(2,3)` R*8 ($\frac{s+m_5-(m_3+m_4)^2}{2W}$) : Maximum of E_5.

2. Physics control section

   - `ICOS3` I*4 (0) : Treatment of θ_3.

3. `BASES` control section

   - `ITMX1` I*4 (5) : These three values control `BASES` integration. See document of `BASES` for details.
   - `ITMX2` I*4 (5) :
   - `NCALL` I*4 (5000) :
   - `NX` I*4 (50) : Value `NX` control the histograms.

A.9 3006

This is the kinematics for radiative processes.

Description of the kinematics

This is the kinematics for 2 to 3 process in the center-of mass system. A particle 3 is assumed to be a photon. To treat both initial and final state radiations, a phase space with respect to photon angles is divided into three regions;

$$\begin{aligned}
\sigma &= \int_S \frac{d\sigma}{d\Omega_3} d\Omega_3, \\
&= \int_{S_{12}} \frac{d\sigma}{d\Omega_3} d\Omega_3 + \int_{S_4} \frac{d\sigma}{d\Omega_3} d\Omega_3 + \int_{S_5} \frac{d\sigma}{d\Omega_3} d\Omega_3, \\
S_{12} &= \{\hat{P}_3 | min\{\theta_{13}, \theta_{23}, \theta_{43}, \theta_{53}\} = \theta_{13} \; or \; \theta_{23}\}, \\
S_4 &= \{\hat{P}_3 | min\{\theta_{13}, \theta_{23}, \theta_{43}, \theta_{53}\} = \theta_{34}\}, \\
S_5 &= \{\hat{P}_3 | min\{\theta_{13}, \theta_{23}, \theta_{43}, \theta_{53}\} = \theta_{35}\},
\end{aligned}$$

where $\theta_{ij} = cos^{-1}(\hat{P}_i \cdot \hat{P}_i)$ and $\hat{P}_i$ is a unit (three) vector along a three momentum of a particle-i.

In the region S_{12}

In the region S_{12}, the kinematics 3003 is used. Difference is quoted here.

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section
   - `COSOPN` R*8 (1) : opening angle cut between photon and particle 4,5
2. Physics control section
   - `IFRAD` I*4 (3) : 1=initial state radiadiation, 2=final state radiation,3=initial+final state radiation.

In the region S_4

In the region S_4, the final state first splits into particle-5 and the system particle-5 and photon (particle-3). After that the latter decays into particle-5 and particle-3.

$$1 + 2 \longrightarrow 5 + q \quad , \quad q \longrightarrow 3(photon) + 5$$

Integration variables are the polar angle θ and azimuthal angle ϕ for the first split, those angles for the second split, and the invariant mass of particles 3 and 5. Angles for the first split are defined with respect to the incoming particles and those for the second split are defined in the center-of-mass system of particles 4 and 5 with respect to the momentum direction of the system.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll} \text{Particle-1} & (0,0,+P,E_1) \\ \text{Particle-2} & (0,0,-P,E_2) \end{array}$$

where P is the positive value determined by m_1, m_2, W. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

The angles θ_4 and ϕ_4 represent the direction of particle-4. Invariant mass of 3 and 5, Q^2 is another variable.

$$Q^2 = (p_3 + p_5)^2$$

In the center of mass system of particles 3 and 5, angles $\theta_{3,(35)}$ and $\phi_{3,(35)}$ represent the direction of particle-3. The system of particles 3 and 5 are boosted backward to the momentum direction of particle-4.

Phase space is in the center-of-mass system and it is given by

$$\begin{aligned} d\tilde{\Gamma}_3 &= \frac{1}{2\pi} d\Gamma_2(1+2 \to 4+q) dQ^2 d\Gamma_2(q \to 3+5) \\ &= \frac{\beta(12;4q)\beta(q;35)}{8^2(2\pi)^5} d\cos\theta_3 d\phi_3 dQ^2 d\cos\theta_{4,(45)} d\phi_{4,(45)}. \end{aligned}$$

To treat colinear singularity, new variables are introduced;

$$\frac{dQ^2}{Q^2} = d(\log Q^2),$$

then,

$$dQ^2 = Q^2 d(\log Q^2) = Q^2 \log(Q^2_{max}/Q^2_{min}) dt$$

where $Q^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^t$ for $0 < t < 1$. Moreover,

$$\begin{aligned} \cos\theta^*_3 &= (\frac{\sqrt{Q^2}E_3}{E^*_3} - E_q)/P_q, \\ E^*_3 &= \frac{Q^2 - m_5^2}{2\sqrt{Q^2}}, \\ E_q &= \frac{s + Q^2 - m_4^2}{2\sqrt{s}}, \\ P_q &= \sqrt{E_q^2 - Q^2}, \end{aligned}$$

where θ_3^* is polar angle of particle-3 in particle-3 and -5 rest frame, E_3 is an energy of particle-3 in a labframe. E_3 is used as integration valiable instead of θ_3^*, then

$$d(\cos\theta_3^*) = \frac{\sqrt{Q^2}}{E_3^* P_q} d(E_3).$$

Futher modification

$$\begin{aligned} \frac{dE_3}{E_3} &= d(\log E_3), \\ &= \log(E_3^{max}/E_3^{min})dt, \end{aligned}$$

for $0 < t < 1$ has been done.

When the flag `icos4=1`, the amplitude is assumed to contain a pole as $1/t^2 = 1/(P_1 - P_4)^2$. The treatment similer to `kinem2002` has been done.

When the flag `icos4=2`, the amplitude is assumed to contain a pole $1/t^2 = 1/(P_1 - P_4)^2$ and $1/u^2 = 1/(P_1 - P_5)^2$. The treatment similer to `kinem2003` has been done.

Meaning of `X()` for BASES integration

`X(1)`	$\cos\theta_4 = 2$`X(1)` $- 1$	`:ICOS4=0`
	$D = -(p1 - p4)^2 + m_1^2$,	
	$D = D_{min}(D_{max}/D_{min})^{X(1)}$	`:ICOS4=1`
	D is symmetrized around X(1)=0.5	`:ICOS4=2`
`X(2)`	$\phi_3 = 2\pi$`X(2)`	
	Azimuthal angle in the CM system of particles 3 and 5.	
`X(3)`	$Q^2 = (P_3 + P_5)^2 = Q^2_{min}(Q^2_{max}/Q^2_{min})^{X(3)}$	
`X(4)`	$E_3 = E_3^{min}(E_3^{max}/E_3^{min})^{X(4)}$	
	Energy of particle-3 (photon)	
`X(5)`	$\phi_4 = 2\pi$`X(5)`	
	Azimuthal angle of particle-4	

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section

   - `W` R*8 (200.0) : Center of mass energy.
   - `COSCUT(1,1)` R*8 (-1.0) : Minimum of $\cos\theta_3$.
   - `COSCUT(2,1)` R*8 (+1.0) : Maximum of $\cos\theta_3$.
   - `COSCUT(1,2)` R*8 (-1.0) : Minimum of $\cos\theta_4$. Here, θ_4 is the polar angle of particle-4 with respect to the beam axis.

- `COSCUT(2,2)` R*8 (+1.0) : Maximum of $\cos\theta_4$.
- `COSCUT(1,3)` R*8 (-1.0) : Minimum of $\cos\theta_5$. Here, θ_5 is the polar angle of particle-5 with respect to the beam axis.
- `COSCUT(2,3)` R*8 (+1.0) : Maximum of $\cos\theta_5$.
- `ENGYCT(1,1)` R*8 (1.D-3) : Minimum of E_3.
- `ENGYCT(2,1)` R*8 ($\frac{s+m_3-(m_4+m_5)^2}{2W}$) : Maximum of E_3.
- `ENGYCT(1,2)` R*8 (m_4) : Minimum of E_4.
- `ENGYCT(2,2)` R*8 ($\frac{s+m_4-(m_5+m_3)^2}{2W}$) : Maximum of E_4.
- `ENGYCT(1,3)` R*8 (m_5) : Minimum of E_5.
- `ENGYCT(2,3)` R*8 ($\frac{s+m_5-(m_3+m_4)^2}{2W}$) : Maximum of E_5.
- `COSOPN)` R*8 (1) : opening angle cut between photon and particle 4,5

2. Physics control section

   - `ICOS4` I*4 (0) : Treatment of θ_4.
   - `IFRAD` I*4 (3) : 1=initial state radiadiation, 2=final state radiation,3=initial+final state radiation.

3. `BASES` control section

   - `ITMX1` I*4 (5) : These three values control `BASES` integration. See document of `BASES` for details.
   - `ITMX2` I*4 (5) :
   - `NCALL` I*4 (5000) :
   - `NX` I*4 (50) : Value `NX` control the histograms.

In the region S_5

$4 \leftrightarrow 5$ of previous section.

A.10 3007

This is the kinematics for double-radiative processes. It is similar to `3006` and only the difference is quoted here

Description of the kinematics

This is the kinematics for 2 to 3 process in the center-of mass system. Particle 3 and 4 are assumed to be photons. A energy ordaring is required;

$$E_3 < E_4$$

A.11 3008

This is the kinematics for three-photon process. It is similar to `3006` and only the difference is quoted here

Description of the kinematics

This is the kinematics for 2 to 3 process in the center-of mass system. Particle 3,4 and 5 are assumed to be photons. A energy ordaring is required;

$$E_3 < E_4 < E_5$$

A.12 3009

This is the kinematics similar to `3001` and only the difference is quoted here.

Description of the kinematics

This kinematics can treat processes in which particle 4 and 5 come from two independent resonances (for example, $b\bar{b}$ from a Higgs boson and a Z-boson). Moreover it can treat some singularities of angluare distribution of particle-4.

In this kinematics, with flag `ICOS4=1or2`, the cross section is assumed to have flat rapidity distrobution. New integration variable η is introduced as

$$-\log(1+2/\epsilon) < \eta < \log(1+2/\epsilon),$$

$$\epsilon = 2m_4^2/(P_4+P_5)^2.$$

By using this variable, $\cos\theta_4$ can be expressed as;

$$\cos\theta_4 = (1+\epsilon)\tanh\eta.$$

$\cos\theta_4$ is measured with respect to $-P_3$.

With flag `ICOST=-1`, the cross section is assumed to have $\sim 1/t$ singularity, where $t = (p1 - p4)^2$. New integration variable is introduced as

$$\begin{aligned} D &= -t, \\ &= 2(E_1 E_4 - P P_4 \cos\theta_4). \end{aligned}$$

$\cos\theta_4$ is measured with respect to P_1. The variable is transformed into

$$\begin{aligned} dD &= 2PP_4 d\cos\theta, \\ dD/D &= d(\log D). \end{aligned}$$

Then the phase space in `3001` is replaced by

$$\begin{aligned} d\cos\theta_4 &= \frac{D}{2PP_4} d(\log D), \\ &= \frac{D}{2PP_4} \log(D_{max}/D_{min}) d\eta \end{aligned}$$

where $D = D_{min}(D_{max}/D_{min})^\eta$ for $0 < \eta < 1$.

With flag `ICOST=-2`, $\cos\theta_4$ is symmetrized around 90°.

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section

   - `ARESNS(1,1)` R*8 (0.0) : Mass of first resonance.
   - `ARESNS(2,1)` R*8 (0.0) : Width of first resonance.
   - `ARESNS(1,2)` R*8 (0.0) : Mass of second resonance.
   - `ARESNS(2,2)` R*8 (0.0) : Width of second resonance.

2. Physics control section

   - `IRESNS` I*4 (0) : Treatment of Q^2. * no-singularity : IRESNS= 0 * narrow resonance (single): IRESNS= 1 * narrow resonance (single) * + 1/Q2 singularit : IRESNS=-1 * narrow resonance (double): IRESNS= 2 * narrow resonance (double) * + 1/Q2 singularit : IRESNS=-2 * Q2=S peak : IRESNS= 3 * 1/Q2 singularity only : IRESNS=-3 * narrow resonance (single) * + Q2=S peak : IRESNS= 4
   - `ICOS4` I*4 (0) : Treatment of θ_4. * no-singularity : ICOS4= 0 * z-axis -¿ -p3 * 1/t singularity : ICOS4= 1 * 1/t + 1/u singularity : ICOS4= 2 * z-axis -¿ p1 * 1/t singularity : ICOS4=-1 * 1/t + 1/u singularity : ICOS4=-2

A.13 4001

Kinematics `4002` is variation of this one and the user can also consult them in the following sections.

Description of the kinematics

This is the kinematics for 2 to 4 process in the center-of mass system. Here the final state first splits into the system of particles 3 and 4 and the system of particles 5 and 6. After that both system decays:

$$1+2 \longrightarrow q_1+q_2 \quad , \quad q_1 \longrightarrow 3+4 \quad , \quad q_2 \longrightarrow 5+6$$

Integration variables are the polar angle θ and azimuthal angle ϕ for the first split, those angles for the two secondary splits, and the invariant masses of q_1 and q_2. Angles for the first split are defined with respect to the incoming particles and those for the secondary splits are defined in their own center-of-mass system with respect to the momentum direction of the system.

If there is a mass singularity for the two of particles in the final state, it is recommended to assign the two particles to form a pair above. Also user can introduce cutoff for angles and minimum energies as options.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll} \text{Particle-1} & (0,0,+P,E_1) \\ \text{Particle-2} & (0,0,-P,E_2) \end{array}$$

where P is the positive value determined by m_1, m_2, W. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

The angles θ_{q1} and ϕ_{q1} represent the direction of the system of particles 3 and 4. Invariant masses are another variables.

$$Q_1^2 = q_1^2 = (p_3+p_4)^2 \quad , \quad Q_2^2 = q_2^2 = (p_5+p_6)^2$$

In the center of mass system of particles 3 and 4, angles $\theta_{3,(34)}$ and $\phi_{3,(34)}$ represent the direction of particle-3. Similarly, $\theta_{5,(56)}$ and $\phi_{5,(56)}$ are defined. The systems of particles 3 and 4,and 5 and 6 are boosted backward to the laboratory frame later.

Phase space is in the center-of-mass system and it is given by

$$\begin{aligned} d\tilde{\Gamma}_4 &= \frac{1}{(2\pi)^8} d\Gamma_2(1+2 \to q_1+q_2) dQ_1^2 dQ_2^2 d\Gamma_2(q_1 \to 3+4) d\Gamma_2(q_2 \to 5+6) \\ &= \frac{\beta(12;q_1q_2)\beta(q_1;34)\beta(q_2;56)}{8^3(2\pi)^8} d\cos\theta_{q1} d\phi_{q1} \\ &\times \; dQ_1^2 d\cos\theta_{3,(34)} d\phi_{3,(34)} dQ_2^2 d\cos\theta_{5,(56)} d\phi_{5,(56)}. \end{aligned}$$

For the treatment of Q_j^2, one can use one of three ways by setting the flags `IRESNS(j), (j=1,2)`.

- Flat distribution to Q_j^2, as in `3001` for the system of particle 4 and 5. The flag is `IRESNS(j)=0`.
- Resonance distribution to Q_j^2, as in `3003` for the system of particle 4 and 5. The flag is `IRESNS(j)=1`. If this mode is selected, also the user should supply the values of mass and width.
- $1/Q_j^2$ distribution, as in `3004` for the system of particle 4 and 5. The flag is `IRESNS(j)=-1`.

Here there is no singular behavior for t channel, and `ICOSQ3=0` is assumed.

Meaning of `X()` for BASES integration

`X(1)`	Invariant mass of particles 3 and 4, Q_1^2. Dependent on `IRESNS(1)`. The same as `X(4)` in `3001`, `3003`, `3004` for `IRESNS(1)=0,1,-1`.
`X(2)`	Invariant mass of particles 5 and 6, Q_2^2. Dependent on `IRESNS(2)`. The same as `X(4)` in `3001`, `3003`, `3004` for `IRESNS(2)=0,1,-1`.
`X(3)`	$\cos\theta_{3,(34)} = 2$`X(3)` $- 1$ Polar angle in the CM system of particles 3 and 4.
`X(4)`	$\phi_{3,(34)} = 2\pi$`X(4)` Azimuthal angle in the CM system of particles 3 and 4.
`X(5)`	$\cos\theta_{5,(56)} = 2$`X(5)` $- 1$ Polar angle in the CM system of particles 5 and 6.
`X(6)`	$\phi_{5,(56)} = 2\pi$`X(6)` Azimuthal angle in the CM system of particles 5 and 6.
`X(7)`	$\cos\theta_{q1} = 2$`X(7)` $- 1$ Polar angle of particle-3
`X(8)`	$\phi_{q1} = 2\pi$`X(8)` Azimuthal angle of q_1

Options in `kinit.f`

Default value is shown in parenthesis.

1. Physical parameter section
   - `W` R*8 (200.0) : Center of mass energy.
   - `COSCUT(1,1)` R*8 (-1.0) : Minimum of $\cos\theta_3$. This and angles below are all in the laboratory frame.

- `COSCUT(2,1)` R*8 (+1.0) : Maximum of $\cos\theta_3$.
- `COSCUT(1,2)` R*8 (-1.0) : Minimum of $\cos\theta_4$.
- `COSCUT(2,2)` R*8 (+1.0) : Maximum of $\cos\theta_4$.
- `COSCUT(1,3)` R*8 (-1.0) : Minimum of $\cos\theta_5$.
- `COSCUT(2,3)` R*8 (+1.0) : Maximum of $\cos\theta_5$.
- `COSCUT(1,4)` R*8 (-1.0) : Minimum of $\cos\theta_6$.
- `COSCUT(2,4)` R*8 (+1.0) : Maximum of $\cos\theta_6$.
- `ENGYCT(1,1)` R*8 (m_3) : Minimum of E_3.
- `ENGYCT(2,1)` R*8 (W) : Maximum of E_3.
- `ENGYCT(1,2)` R*8 (m_4) : Minimum of E_4.
- `ENGYCT(2,2)` R*8 (W) : Maximum of E_4.
- `ENGYCT(1,3)` R*8 (m_5) : Minimum of E_5.
- `ENGYCT(2,3)` R*8 (W) : Maximum of E_5.
- `ENGYCT(1,4)` R*8 (m_6) : Minimum of E_6.
- `ENGYCT(2,4)` R*8 (W) : Maximum of E_6.
- `AMASCT(1,1)` R*8 ($m_3 + m_4$) : Minimum of Q_1. Q_1 is the mass of the system of particles 3 and 4.
- `AMASCT(2,1)` R*8 ($W - m_5 - m_6$) : Maximum of Q_1.
- `AMASCT(1,2)` R*8 ($m_5 + m_6$) : Minimum of Q_2. Q_2 is the mass of the system of particles 5 and 6.
- `AMASCT(2,2)` R*8 ($W - m_3 - m_4$) : Maximum of Q_2.
- `ARESNS(1,1)` R*8 (0.0) : Mass of resonance, this and the next parameter are meaningful only when `IRESNS(1)=+1`.
- `ARESNS(2,1)` R*8 (0.0) : Width of resonance.
- `ARESNS(1,2)` R*8 (0.0) : Mass of resonance, this and the next parameter are meaningful only when `IRESNS(2)=+1`.
- `ARESNS(2,2)` R*8 (0.0) : Width of resonance.

2. Physics control section

- `NDIM` I*4 (8) : If you want to suppress ϕ_{q1}-integration, replace the definition of `NDIM` by the following:

  ```
  NDIM = 7
  ```

  Then ϕ_{q1} is fixed to 0.0.
- `IRESNS(1)` I*4 (0) : Treatment of Q_1^2. See the description of kinematics for the meaning.

- IRESNS(2) I*4 (0) : Treatment of Q_2^2. See the description of kinematics for the meaning.
- ICOSQ3 I*4 (0) : Treatment of θ_{q1}. (Do not change.)

3. BASES control section

- ITMX1 I*4 (5) : These three values control BASES integration. See document of BASES for details.
- ITMX2 I*4 (5) :
- NCALL I*4 (5000) :
- NX I*4 (50) : Value NX control the histograms.

Related modules

WTOLAB

A.14 4002

This is the kinematics similar to 4001 and only the difference is quoted here.

Description of the kinematics

In this kinematics, with flag ICOSQ3=1 , new integration variable is introduced as

$$\begin{aligned} T &= -(p_1 - Q_1)^2, \\ &= 2(E_1 E_{q_1} - P P_{q_1} \cos\theta_{q_1}) - m_1^2 - Q_1^2. \end{aligned}$$

Moreover with flag ICOSQ3=2 , the cross section is assumed to have $\sim 1/T$ singularity. The variable is transformed into

$$\begin{aligned} \mathrm{dT} &= 2PP_{q_1} d\cos\theta_{q_1}, \quad \text{(for ICOSQ3=1)} \\ \mathrm{dT/T} &= d(\log T). \quad \text{(for ICOSQ3=2)} \end{aligned}$$

Then the phase space in 4001 is replaced by

$$\begin{aligned} d\cos\theta_{q_1} &= \tfrac{1}{2PP_{q_1}} d\cos\theta_{q_1}, \quad \text{(for ICOSQ3=1)} \\ &= \tfrac{T}{2PP_{q_1}} \log(T_{max}/T_{min}) d\eta \quad \text{(for ICOSQ3=2)} \end{aligned}$$

where $T = T_{min}(T_{max}/T_{min})^{\eta}$ for $0 < \eta < 1$.

Meaning of X() for BASES integration

Except for X(7), they are the same in 4001.

X(7) Momentum transfer square, $T = -(p_1 - Q_1)^2$.
$T = T_{min} + (T_{max} - T_{min})$X(7) (for ICOSQ3=1)
$T = T_{min}(T_{max}/T_{min})^{\mathrm{X(7)}}$ (for ICOSQ3=2)

Options in `kinit.f`

Physics control section

- `ICOSQ3` I*4 (1) : Treatment of θ_{q1}. (Do not change.)

A.15 4003

Description of the kinematics

This is the kinematics for 2 to 4 process in the center-of mass system. Here a particle 3 emmit particle A and a particle 4 emmit particle B. After that particles A and B collide into particles 5 and 6;

$$1 \longrightarrow 3 + A \quad , \quad 2 \longrightarrow 4 + B \quad , \quad A + B \longrightarrow 5 + 6$$

Integration variables are the polar angle θ and azimuthal angle ϕ of particles 3 and 4, those angles of particles 5 and 6 in their rest frame, energies of particles 3 and 4, and the invariant masses of 5 and 6. Angles of 3 and 4 are defined with respect to the incoming particles and those of 5 and 6 are defined in their own center-of-mass system with respect to the momentum direction of the system.

User can introduce cutoff for angles and minimum energies as options.

For the 4-vector notation, we use (p_x, p_y, p_z, E) ordering. We use the frame where incoming particles collide along z-axis, so that the momenta of initial particles are assigned as follows:

$$\begin{array}{ll} \text{Particle-1} & (0, 0, +P, E_1) \\ \text{Particle-2} & (0, 0, -P, E_2) \end{array}$$

where P is the positive value determined by m_1, m_2, W. Here, relative velocity v_{rel} is assumed to be 2 for the collision highly relativistic particles. If colliding particles are slow, the user needs modification in `kinit.f`.

New variables are introduces as ;

$$\begin{aligned} Q_{1,2}^2 = -q_{1,2}^2 &= -(p_1 - p_{3,4})^2, \\ &= 2(EE_{3,4} \mp P_{1,2}P_{3,4}\cos\theta_{3,4}), \end{aligned}$$

and

$$q_{1,2}^0 = E - P_{3,4}.$$

Invariant mass is another variables.

$$Q_3^2 = q_3^2 = (p_5 + p_6)^2.$$

Phase space is in the center-of-mass system and it is given by

$$\begin{aligned} d\tilde{\Gamma}_4 &= \frac{1}{16(2\pi)^6}\frac{1}{P_3P_4}\frac{1}{\sqrt{\Omega^2-\Psi^2}}dQ_1^2dQ_2^2dq_1^0dq_2^0d\phi_3 \\ &\times \beta(q_1q_2:56)dQ_1^2d\cos\theta_{5,(56)}d\phi_{6,(56)}, \end{aligned}$$

where

$$\begin{aligned} \Psi &= W^2-2W(P_3+P_4)+2(E_3E_4-P_3P4\cos\theta_3\cos\theta_4) \\ &\quad +m_3^2+m_4^2-Q_3^2, \\ \Omega &= 2E_3E_4\sin\theta_3\sin\theta_4. \end{aligned}$$

For the treatment of Q_3^2, one can use one of three ways by setting the flags `IRESNS`.

- Flat distribution to Q_3^2, as in `3001` for the system of particle 4 and 5. The flag is `IRESNS=0`.
- Resonance distribution to Q_3^2, as in `3003` for the system of particle 4 and 5. The flag is `IRESNS=1`. If this mode is selected, also the user should supply the values of mass and width.
- $1/Q_3^2$ distribution, as in `3004` for the system of particle 4 and 5. The flag is `IRESNS=1`.

When the flag `ICOS3,4=1`, the kinematics is changed to the case for the radiation of particle-3,4 along beam axis.

$$dQ_{1,2}^2/Q_{1,2}^2 = d(\log Q_{1,2}^2).$$

Meaning of X() for BASES integration

X(1)	Energy of $q_1 = E_1 - E_3$
X(2)	Energy of $q_2 = E_2 - E_4$
X(3)	Momentun transfer squaer, $Q_1^2 = -(p_1 - p_3)^2$ $Q_1^2 = Q_{1,min}^2 + (Q_{1,max}^2 - Q_{1,min}^2)X(3)$ for ICOS3=0 $Q_1^2 = Q_{1,min}^2 (Q_{1,max}^2/Q_{1,min}^2)^{X(3)}$ for ICOS3=1
X(4)	Momentun transfer squaer, $Q_2^2 = -(p_2 - p_4)^2$ $Q_2^2 = Q_{2,min}^2 + (Q_{2,max}^2 - Q_{2,min}^2)X(4)$ for ICOS4=0 $Q_2^2 = Q_{2,min}^2 (Q_{2,max}^2/Q_{2,min}^2)^{X(4)}$ for ICOS4=1
X(5)	$\cos\theta_{5,(56)} = 2$X(5) $- 1$ Polar angle in the CM system of particles 5 and 6.
X(6)	$\phi_{5,(56)} = 2\pi$X(6) Azimuthal angle in the CM system of particles 5 and 6.
X(7)	Invariant mass of particles 5 and 6, Q_3^2. Dependent on IRESNS . The same as X(4) in 3001, 3003, 3004 for IRESNS(2)=0,1,-1.
X(8)	$\phi_3 = 2\pi$X(8) Azimuthal angle of particle-3

Options in kinit.f

Default value is shown in parenthesis.

1. Physical parameter section

   - W R*8 (200.0) : Center of mass energy.
   - COSCUT(1,1) R*8 (-1.0) : Minimum of $\cos\theta_3$. This and angles below are all in the laboratory frame.
   - COSCUT(2,1) R*8 (+1.0) : Maximum of $\cos\theta_3$.
   - COSCUT(1,2) R*8 (-1.0) : Minimum of $\cos\theta_4$.
   - COSCUT(2,2) R*8 (+1.0) : Maximum of $\cos\theta_4$.
   - COSCUT(1,3) R*8 (-1.0) : Minimum of $\cos\theta_5$.
   - COSCUT(2,3) R*8 (+1.0) : Maximum of $\cos\theta_5$.
   - COSCUT(1,4) R*8 (-1.0) : Minimum of $\cos\theta_6$.
   - COSCUT(2,4) R*8 (+1.0) : Maximum of $\cos\theta_6$.
   - ENGYCT(1,1) R*8 (m_3) : Minimum of E_3.
   - ENGYCT(2,1) R*8 (W) : Maximum of E_3.
   - ENGYCT(1,2) R*8 (m_4) : Minimum of E_4.
   - ENGYCT(2,2) R*8 (W) : Maximum of E_4.

- `ENGYCT(1,3)` R*8 (m_5) : Minimum of E_5.
- `ENGYCT(2,3)` R*8 (W) : Maximum of E_5.
- `ENGYCT(1,4)` R*8 (m_6) : Minimum of E_6.
- `ENGYCT(2,4)` R*8 (W) : Maximum of E_6.
- `AMASCT(1)` R*8 ($m_5 + m_5$) : Minimum of Q_3. Q_1 is the mass of the system of particles 3 and 4.
- `AMASCT(2)` R*8 ($W - m_3 - m_4$) : Maximum of Q_3.
- `ARESNS(1)` R*8 (m_W) : Mass of resonance, this and the next parameter are meaningful only when `IRESNS(1)=+1`.
- `ARESNS(2)` R*8 (Γ_W) : Width of resonance.

2. Physics control section

   - `NDIM` I*4 (8) : If you want to suppress ϕ_{q1}-integration, replace the definition of `NDIM` by the following:

     ```
     NDIM = 7
     ```

     Then ϕ_3 is fixed to 0.0.
   - `IRESNS` I*4 (0) : Treatment of Q_3^2. See the description of kinematics for the meaning.
   - `ICOS3` I*4 (1) : Treatment of θ_3.
   - `ICOS4` I*4 (0) : Treatment of θ_4.

3. `BASES` control section

   - `ITMX1` I*4 (5) : These three values control `BASES` integration. See document of `BASES` for details.
   - `ITMX2` I*4 (5) :
   - `NCALL` I*4 (5000) :
   - `NX` I*4 (50) : Value `NX` control the histograms.

Related modules

`PBOOST ROXMTX MINVR2 MVMULT`

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[3] CERN CN Division, KUIP, CERN Program Library Long Writeup I102, October 1994.

[4] CERN CN Division, PAW++, CERN Program Library Long Writeup Q121, September 1993.

[5] H. Tanaka, Comput. Phys. Commun. **58**, 153 (1990).

[6] S. Kawabata, Comput. Phys. Commun. **88**, 309 (1995).