Systematic evaluation

of the MSSM spectrum

Vincent LAFAGE

***Minami-Tateya Group, IPNS, KEK,* TSUKUBA**

The MSSM Spectrum & its evaluation

- Motivation
- The MSSM parameters & RGE
- The ODE & integration: results
- Thresholds
- Other radiative corrections
- 2 boundary conditions
- Program structure
  MUSE: a MUltiscale Spectrum Evaluator
- Results

Motivation

- **SUSY has so many nice features...**
  - **solution to the hierarchy problem**
  - **allows Grand Unification**
  - **link with gravitation**

- **... and so many problems!**
  - **118 free parameters**
    (including ∼40 CP violating phases)
    to be compared with the 19 of the Standard Model.
  - **where do the breaking come from?**
  - **we know only half of the spectrum ☺**
  - **any analysis/limit needs a lot of assumptions**

$\Longrightarrow$ **mSUGRA paradigm:**

- **minimal:** $M_{1/2}$, m_0^2, A_0, $\tan\beta$, $\mathrm{sgn}\,\mu$ @ Λ_{GUT}

- **... require a lot of effort to extract the spectrum**
  `Isajet`, `Spythia`, `SuSpect`...

MSSM parameters

- **gauge couplings,** g_i
- **superpotential: Yukawa's couplings** y_f **complex matrices in family space, mixing angles and phase and** μ * **in principle**
- **soft breaking terms:**
  - **the gaugino masses,** M_i
  - **the trilinear parameters,** a_f **(matrices in family space, but "aligned" with Yukawa's matrices) and the** b **mass term.**
  - **the scalar (higgs / sfermion) square masses** m^2**: also matrices in family space. but set proportional to identity to evade both FCNC and CP-violation.**

RGE
Renormalisation Group Equation

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = \beta_i(y_j, \{t\})$$

β	g	y	μ	M	a	b	m^2
β_g	$\times$	$\circ$					
β_y	$\times$	$\times$					
β_μ	$\times$	$\times$	$\times$				
β_M	$\times$	$\circ$		$\times$	$\circ$		
β_a	$\times$	$\times$		$\times$	$\times$		
β_b	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	
β_{m^2}	$\times$	$\times$		$\times$	$\times$		$\times$

$\times$**: dependency starting at 1-loop**
$\circ$**: dependency appearing at higher order**

Non-linear but polynomial $\Longrightarrow$ very-smooth (between thresholds)

*and R-parity breaking terms...

RGE
basic phenomenology

β	g	y	μ	M	a	b	m^2
β_g	$(+,+,-)$	$\circ$					
β_y	$-$	$+$					
β_μ	$-$	$+$	$\times$				
β_M	$(+,+,-)$	$\circ$		$\times$	$\circ$		
β_a	$\times$	$\times$		$\times$	$\times$		
β_b	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	
β_{m^2}	$-$	$+$		$-$	$+$		$+$

$\times$: dependency starting at 1-loop
$\circ$: dependency appearing at higher order

- **gauge unification**
- **radiative EW breaking**
- μ **and** b **interplay only together**
- g **and** y **are driven only by** g **and** y
- $g_3 \sim 1.22$ **and** $y_t \sim 1.05$ **are the most efficient drivers**
- y_t **can exhibit infrared fixed point (**$y_t \sim 1.25$**)**

ODE integration
Results

Average number of derivative evaluations to get accuracy of ϵ (integrating one-loop β functions between M_Z and Λ_{GUT}).

ϵ	Step Doubling	CASH KARP	BULIRSCH STOER (polynomial)	BULIRSCH STOER (rational)	Predictor Corrector
5.10^{-4}	277	79	107	55	18
5.10^{-8}	507	130	136	110	42
5.10^{-12}	3221	742	225	226	100
5.10^{-16}	21227	4706	607	10270	204

— `Isajet` **starts with 800 derivative evaluations (but no error control and only 4th order).**

— **ODE integration is the core element of the computation: precision of other elements cannot be better.**

— **check effects of higher number of loops (without loosing them in numerical fluctuations).**

— **comparison with analytical solution In fact, not much to gain, neither in speed, nor in accuracy.**

$$y_t = \frac{y_t^0 E_{12}(t)}{1 - b y_t^0 \int_0^t E_{12}(t')dt'} \qquad y_b = \frac{y_b^0 E_{21}(t)}{1 - b y_b^0 \int_0^t E_{21}(t')dt'} \tag{1}$$

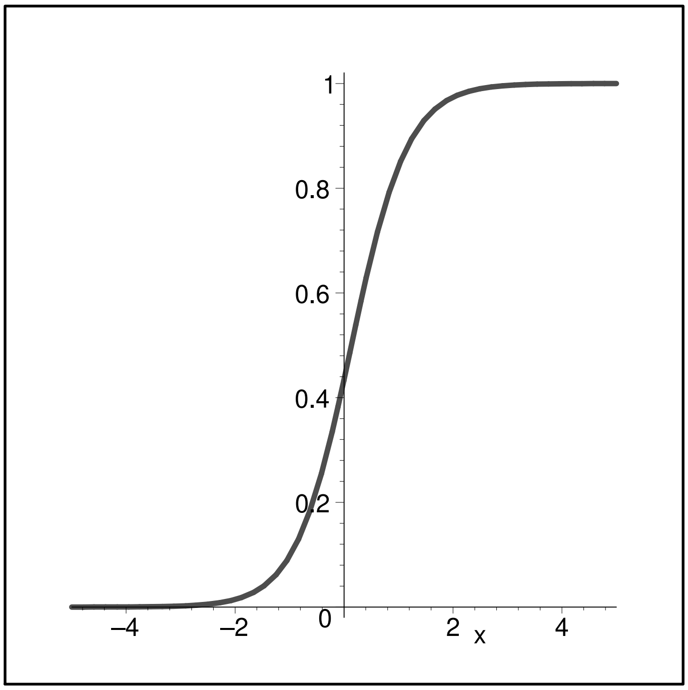
$$E_i = e^{\int_0^t f_i(t')dt'} \tag{2}$$

$$E_{12}(t) = \frac{E_1(t)}{\left(1 - b y_b^0 \int_0^t \frac{E_2(t_1)dt_1}{\left(1 - b y_t^0 \int_0^{t_1} \frac{E_1(t_2)dt_2}{\left(1 - b y_b^0 \int_0^{t_2} \frac{E_2(t_3)dt_3}{\left(1 - b y_t^0 \int_0^{t_3} \ldots\right)^{a/b}}\right)^{a/b}}\right)^{a/b}}\right)^{a/b}} \tag{3}$$

Thresholds

- **Minimal substraction schemes ($\overline{\text{MS}}$ and $\overline{\text{DR}}$)**

$\Longrightarrow$ easier for computation:
β function have no explicit scale dependence ($\neq$ On-Shell (OS) scheme)



OS scheme

reproduce two effective theories with different β functions after and before the threshold
$\Longrightarrow$ HEAVISIDE step function θ (0-width threshold).
(1-loop threshold $\sim$ 2-loop effect; 1-loop threshold width $\sim$ 3-loop effect)

- **top quark threshold.**

- **one common SUSY threshold.**

  – **usual approximation** (as the width blurr them all in one)

  – $\Lambda_{SUSY} \sim 1$ **TeV?**

  – $\Lambda_{SUSY} \sim \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$**?**

  – $\Lambda_{SUSY} = |\mu| \left(\frac{M_2}{M_3}\right)^{\frac{28}{19}} \left(\frac{M_A}{\mu}\right)^{\frac{3}{19}} \left(\frac{M_2}{\mu}\right)^{\frac{4}{19}} \prod_i m_{\tilde{u}_{i2}}^{\frac{-15}{114}} m_{\tilde{d}_{i2}}^{\frac{1}{2}} m_{\tilde{u}_{i1}}^{\frac{-15}{57}} m_{\tilde{d}_{i1}}^{\frac{-9}{57}} m_{\tilde{\ell}_{i2}}^{\frac{7}{38}} m_{\tilde{\ell}_{i1}}^{\frac{-2}{19}} m_{\tilde{\nu}_i}^{\frac{-1}{38}}$
  $\sim |\mu|/5 < M_Z$

Other radiative corrections

• **Running masses are renormalisation scale dependent**
$\Longrightarrow$ **unphysical, not observable**
$\Longrightarrow$ **we need to compute the pole mass** (pole of propagator)
(good observable: gauge invariant, renormalisation scale independent, infrared stable)

$$m_t \quad = \quad \hat{m}_t(Q) - \Re e\left(\Sigma_t(m_t, Q)\right), \qquad Q \sim m_t$$

Self-energy need the whole spectrum to be computed.

• **Two contributions :**

$$\begin{aligned} m_t = \hat{m}_t(Q = m_t) \quad - \quad & \Sigma(Q = m_t) \\ & \propto \ln(Q/m_t) + \textbf{next-to-leading part} \end{aligned}$$

– **"running" leading log**

– **next-to-leading part of self-energy**

For the top, there is a large next-to-leading contribution (-15 GeV).
For the stops, both contribution are important.

• **Higgs spectrum radiative corrections (Carena & Wagner, Weiglein)**

• **Radiative correction in SUSY are computed with dimensional reduction: $\overline{\text{DR}}$**
same β functions, but different finite parts than $\overline{\text{MS}}$

$$\alpha_s|_{\overline{\textbf{DR}}}(M_Z) = \frac{\alpha_s|_{\overline{\textbf{MS}}}(M_Z)}{1 - \Delta\alpha_s}$$

$\Longrightarrow$ **-15% to -20% effect! No other program takes it into account!**

2 boundary conditions
High energy scale

- **Unification**

$$\begin{aligned} g_1(\Lambda_{GUT}) &= g_\infty \\ g_2(\Lambda_{GUT}) &= g_\infty \end{aligned}$$

- **common gaugino mass**

$$M_{1/2} = M_1 = M_2 = M_3$$

- **common scalar mass**

$$\begin{aligned} m_0^2 &= \tilde{m}_e^2 = \tilde{m}_\ell^2 \\ &= \tilde{m}_q^2 = \tilde{m}_u^2 = \tilde{m}_d^2 \\ &= \tilde{m}_{H_u}^2 = \tilde{m}_{H_d}^2 \end{aligned}$$

- **common trilinear coupling**

$$A_0 = A_t = A_b = A_\tau$$

2 boundary conditions
Low energy conditions

- **Unification**

$$\widehat{\alpha} = \frac{\alpha_{em}}{1 - \Delta\widehat{\alpha}} \quad ; \quad \sin^2\widehat{\theta}\cos^2\widehat{\theta} = \frac{\pi\alpha_{em}}{\sqrt{2}M_Z^2 G_\mu (1 - \Delta\widehat{r})}$$

- **Yukawa's coupling**

$$\begin{aligned} y_t(m_t) &= \frac{\sqrt{2}\widehat{m}_t(m_t)}{v(m_t)\sin\beta(m_t)} \\ \text{with} \quad \widehat{m}_t(m_t) &= m_t + \Re e\left(\Sigma_t(m_t, Q = m_t)\right) \\ \text{and} \quad v^2(Q) &= 4\frac{\widehat{M}_Z^2(Q)}{g^2(Q) + g^2(Q)} \end{aligned}$$

- **Electroweak symmetry breaking**

$$\begin{aligned} \tan\beta(|\mu|^2 + \bar{m}_{H_u}^2) &= b + \frac{\widehat{M}_Z^2}{2}\cos(2\beta)\tan\beta \\ \cot\beta(|\mu|^2 + \bar{m}_{H_d}^2) &= b - \frac{\widehat{M}_Z^2}{2}\cos(2\beta)\cot\beta \\ \bar{m}_{H_u}^2 = m_{H_u}^2 - \frac{t_u}{v_u} & \qquad \bar{m}_{H_d}^2 = m_{H_d}^2 - \frac{t_d}{v_d} \\ \widehat{M}_Z^2(Q) &= M_Z^2 + \Re e\left(\Pi_{ZZ}^T(M_Z^2, Q)\right) \end{aligned}$$

Program structure

SM parameters
MSSM assumptions

- parameters set (from file)

$\Downarrow$

RGE $M_Z \to \Lambda$ — load for initial shoot

$\Downarrow$

- entry point for new load

RGE $\Lambda \to m_t \to M_Z \to m_b$

$\Downarrow$

pole mass correction

tadpoles
(potential minimisation)
self-energy
Higgs effective potential

$\Downarrow$

fits at m_t, M_Z, m_b**?**

- No: modify load and $\Uparrow$
- Yes: output results

Solving constraints

$(\Lambda_{GUT}, g_\infty, y_t, y_b, y_\tau, \mu, b)(\Lambda_{GUT})$
$\to (g_1 \& g_2(M_Z), y_t(m_t), y_b(m_b), y_\tau(m_\tau), \mu \& b(\Lambda_{EWSB}))$
$\to$ **tree-level spectrum at low energy**
$\Longrightarrow \Pi^T_{ZZ}(M_Z^2, Q), \dfrac{t_u}{v_u}, \dfrac{t_d}{v_d}, \Delta\hat{\alpha}, \Delta\hat{r}, \cdots$
$\to (g_1(M_Z)_0, g_2(M_Z)_0, y_t(m_t)_0, y_b(m_b)_0, y_\tau(m_\tau)_0,$
$\mu(\Lambda_{EWSB})_0, b(\Lambda_{EWSB})_0)$

First idea: feed back:
run-up of the low energy reference values

Bad idea:
the RGE are very sensitive to low energy initial condition

The top-down approach gives less integration problems.

Solve simultaneously for all the constraints: $\Leftrightarrow$ minimise

$$|g_{1RGE} - g_{10}|^2 + |g_{2RGE} - g_{20}|^2 + \cdots$$

we are looking for a null minimum:

- **Newton with global convergence**
- **Powell Hybrid Method (`MinPack`)**
- **Simplex (slow but robust)**

Parameters management

input parameters

```
30                      'number of parameters'
26                      'number of equations'
100                     'ODE relative size of initial step'
1d-14                   'ODE integration required accuracy'
1d-3                    'ODE smallest step allowed as fraction of initial step'
1d-12                   'NLE TOLF='
1d-12                   'NLE TOLMIN='
1d-08                   'NLE TOLX='
100d0                   'NLE STPMX='
4                       'ODE method : 1=CK, 2=SD, 3=BS, 4=PC, 5=PCS'
.t.                     'polynomial extrapolation in BS method'
.t.                     'uses MinPack routines ?'
2                       'loop(s) for gauge  couplings RGE'
1                       'loop(s) for yukawa couplings RGE'
1                       'loop(s) for soft breaking RGE'
.t.                     'uses thresholds in RGE'
.t.                     'computed threshold ?'
.t.                     'mSUGRA ?'
.t.                     'unification at Lambda'
.t.                     'gauge trinification'
.t.                     'convert alpha strong from MSbar to DRbar'
.t.                     'EW breaking enforcement'
.t.                     'one-loop potential minimisation'
.t.                     'Higgs correction by Carena et al.'
2.1117d16               'Scale Lambda'
100d0                   'SUSY common threshold'
8d0                     'scaling for conditioning'
.t.                     'loud report'
.t.                     'print flag for score'
0                       'print flag for LSODE'
'(I6,G11.5,26G10.3)'    'print format'
```

Parameters management

standard parameters

```
91.187d0            'Z^0 pole mass'
1.16639d-5          'Fermi constant'
137.0359895d0       '1/a_em(0)'
128.8d0             '1/a_em(M_Z)'
0.12d0              'a_S'
.51099907d-3        'e pole mass'
105.658389d-3       '\mu pole mass'
1.777d0             '\tau pole mass'
.010d0              'd mass'
.004d0              'u mass'
.200d0              's mass'
1.3d0               'c mass'
4.89d0              'b mass'
175d0               't pole mass'
```

SUSY breaking assumptions

```
2d0                 '\tan\beta'
444d0               '\mu'
150d0               'm1/2'
300d0               'm0'
-600d0              'A0'
```

Parameters management output

```
out.spc

...
chargino1            107.66955064304521272  75
chargino2            466.12289127035376168  76
neutralino1           56.83499142802314452  71
neutralino2          109.02839446529753786  72
neutralino3          449.68261374373014405  73
neutralino4          469.11173446756401972  74
gluino               428.30290439269373337  70
selectron1           307.48738441870898441  61
selectron2           321.09275972257506737  62
snu-e                315.06471768427297775  63
...
```

```
out.aux

\tan\beta              2.00000000000000
\mu                    448.168673644143
vev(M_Z)               245.194375919816
\Lambda               0.242177221990288E+17
\alpha_{GUT}          0.407947175675616E-01
\sin^2\theta          0.235684350946388
\alpha_s              0.115540647458758
\alpha_s(M_Z)         0.115540647458758
\alpha(M_Z)           0.776397515527953E-02
m_t(M_Z)               163.065532865848
m_H_u^2(M_Z)          -135555.457473235
m_H_d^2(M_Z)           94995.0904390177
M1                     61.9008225842594
M2                     123.391684032895
M3                     424.836796335498
A_t                   -337.905373326048
A_b                   -2128.49188237005
A_\tau                -1006.43784575147
\theta_~t              1.00353967957185
\theta_~b             0.153773926998007
\theta_~\tau           1.24153202869366
\theta_~c              1.44791435823368
```

Comparison

DPF Point 1

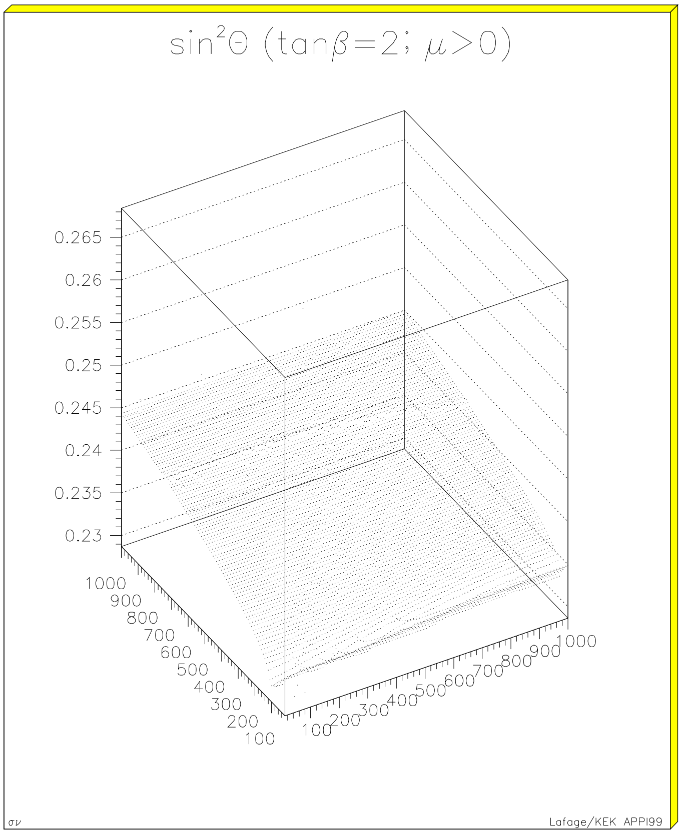
$$\mathbf{m_0 = 400} \quad \mathbf{M_{\frac{1}{2}} = 200} \quad \mathbf{A_0 = 0} \quad \tan\beta = \mathbf{2} \quad \mathrm{sgn}(\mu) = -$$

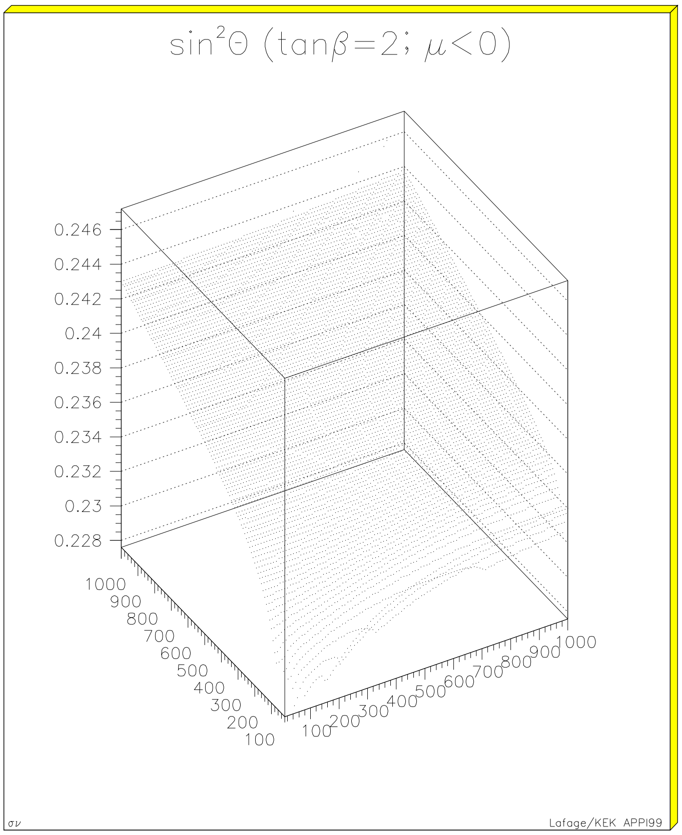
	Muse1	Muse2	Muse2′	Muse2″	Isajet	Suspect	Spythia
α_S	0.1150	0.1258	0.1254	0.1314	0.1314	0.1049	0.1254
Λ_{GUT}	1.2729	1.9656	1.9089	1.8951	2.1117	2.7465	
α_{GUT}	0.0393	0.0409	0.0408	0.0408	0.0406	0.0417	0.041
A_{top}	−326.2	−361.4	−378.2	−403.4	−360.9	−368.2	−337.9
μ	−490.7	−480.8	−485.8	−480.8	−491.7	−504.9	−521.6
$\tilde{l}_1$	408.9	408.8	409.0	409.0	408.8	408.8	408.8
$\tilde{l}_2$	426.9	426.1	427.8	427.9	425.9	426.4	426.8
$\tilde{\nu}$	422.4	421.6	423.3	423.4	421.4	421.9	422.2
$\tilde{\tau}_1$	408.5	408.4	408.6	408.6	408.4	408.4	408.8
$\tilde{\tau}_2$	427.0	426.2	427.9	428.0	426.0	426.4	426.8
$\tilde{\nu}_\tau$	422.3	421.5	423.2	423.3	421.3	†	422.2
$\tilde{\chi}_1^0$	87.6	81.7	85.5	85.5	86.1	85.6	84.4
$\tilde{\chi}_2^0$	173.3	159.3	172.4	172.4	175.7	172.5	169.9
$\tilde{\chi}_3^0$	502.7	492.5	497.3	492.0	494.2	507.1	523.9
$\tilde{\chi}_4^0$	511.7	501.8	506.2	501.0	503.2	516.3	532.7
$\tilde{\chi}_1^\pm$	173.3	159.3	172.4	172.4	175.6	172.3	169.8
$\tilde{\chi}_2^\pm$	510.8	500.9	505.4	500.2	502.3	515.3	531.8
h_0	78.1	79.1	79.6	80.4	80.2	77.3	77.4
H_0	729.0	717.8	722.3	716.6	729.4	744.3	750.5
A_0	724.7	713.4	717.9	712.2	724.5	740.0	745.6
$H^\pm$	728.9	717.7	722.2	716.5	729.1	744.1	753.7
$\tilde{u}_1$	570.0	582.1	590.4	598.7	596.6	608.9	574.6
$\tilde{u}_2$	582.5	593.9	602.9	611.0	609.0	621.4	586.7
$\tilde{d}_1$	569.2	581.4	589.7	598.0	595.9	608.1	573.9
$\tilde{d}_2$	585.7	597.0	606.0	614.0	612.2	624.4	589.7
$\tilde{t}_1$	403.6	428.3	437.1	451.2	421.0	429.2	390.6
$\tilde{t}_2$	533.7	553.4	563.6	577.4	546.8	557.4	516.9
$\tilde{b}_1$	489.9	506.5	515.9	526.9	523.4	532.8	492.4
$\tilde{b}_2$	569.2	581.3	589.7	598.0	595.7	607.9	573.8
$\tilde{g}$	517.0	525.6	548.3	561.2	553.5	524.3	488.3

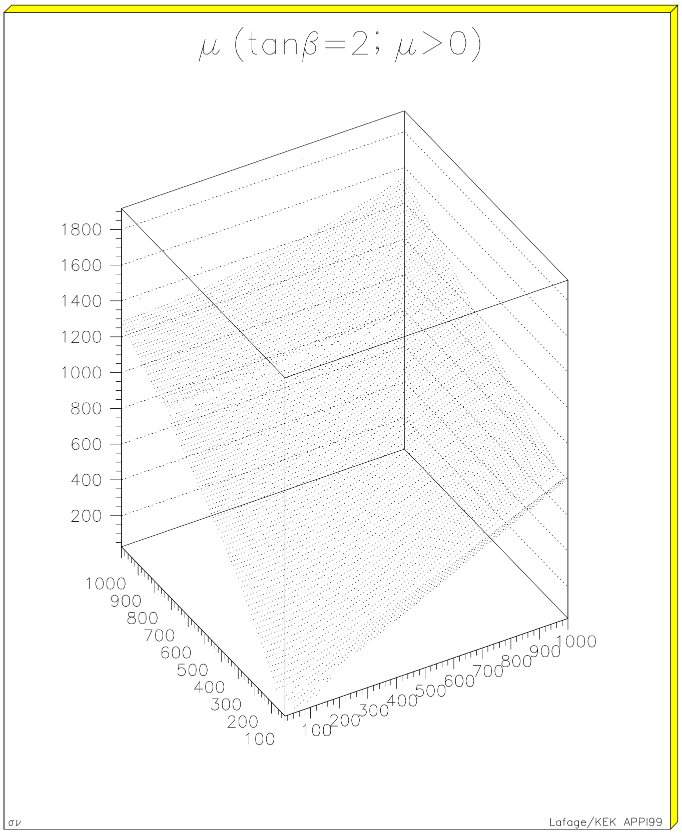
First comparisons

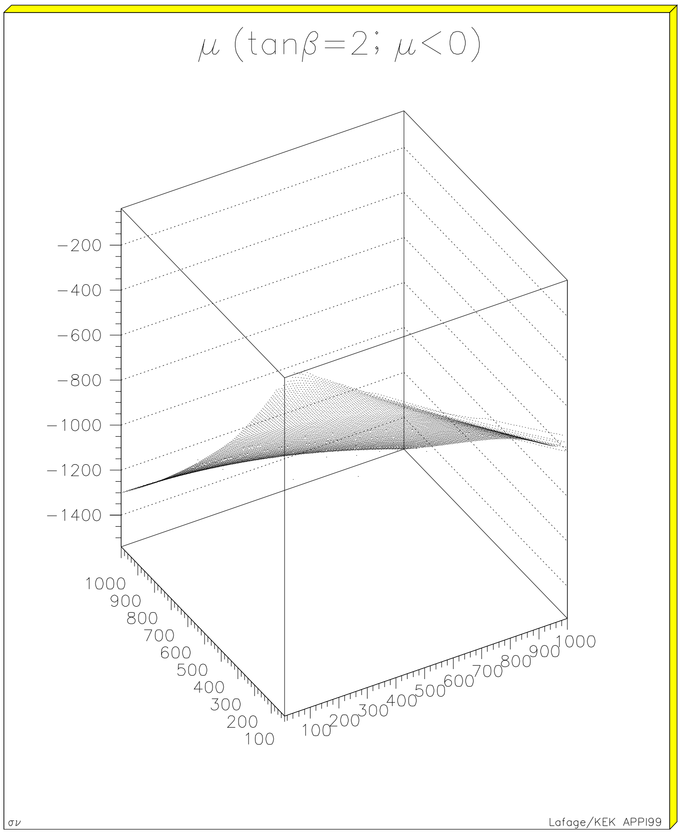
- Very good agreement on sleptons
  (not so much scale dependence)
- agreement on light charginos / neutralinos
- disagreement on heavy charginos / neutralinos
  $\Longrightarrow \mu$ and potential minimisation
- disagreement on squarks
- disagreement on higgs
- disagreement on gluino
- role of running
- 5% on α_s gives 2.5% on scalars running mass.

Scanning

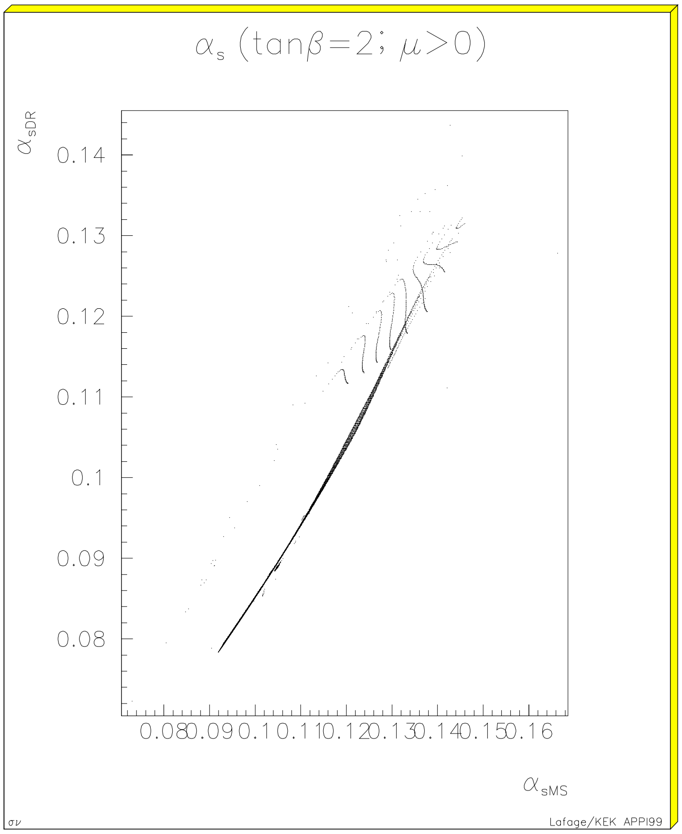

sin$^2\Theta$ (tanβ=2; μ>0)
0.265
0.26
0.255
0.25
0.245
0.24
0.235
0.23
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
$\sigma\nu$
Lafage/KEK APPI99


sin$^2\Theta$ (tanβ=2; μ<0)
0.246
0.244
0.242
0.24
0.238
0.236
0.234
0.232
0.23
0.228
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
$\sigma\nu$
Lafage/KEK APPI99

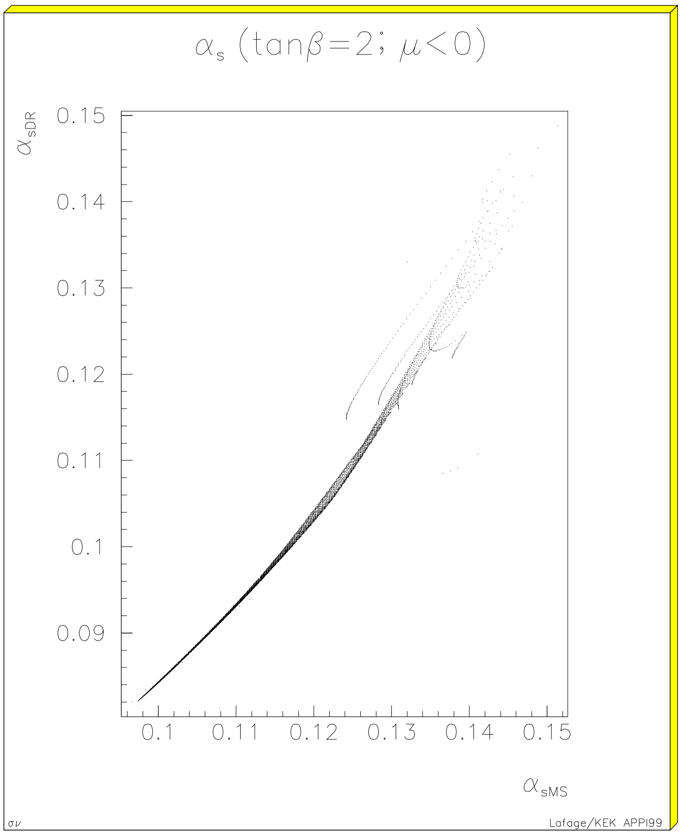

μ (tanβ=2; μ>0)
1800
1600
1400
1200
1000
800
600
400
200
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
$\sigma\nu$
Lafage/KEK APPI99


μ (tanβ=2; μ<0)
−200
−400
−600
−800
−1000
−1200
−1400
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
$\sigma\nu$
Lafage/KEK APPI99

α_S

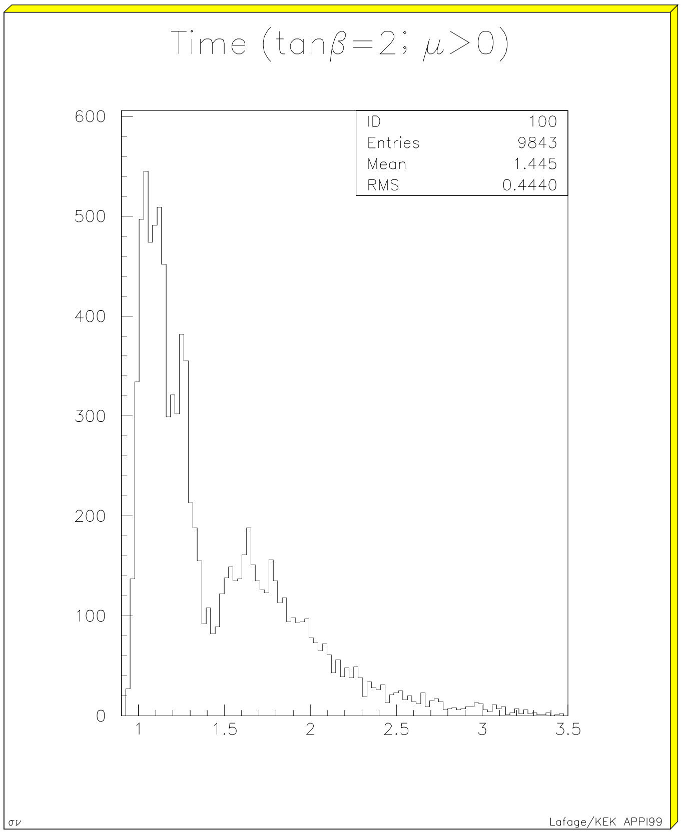


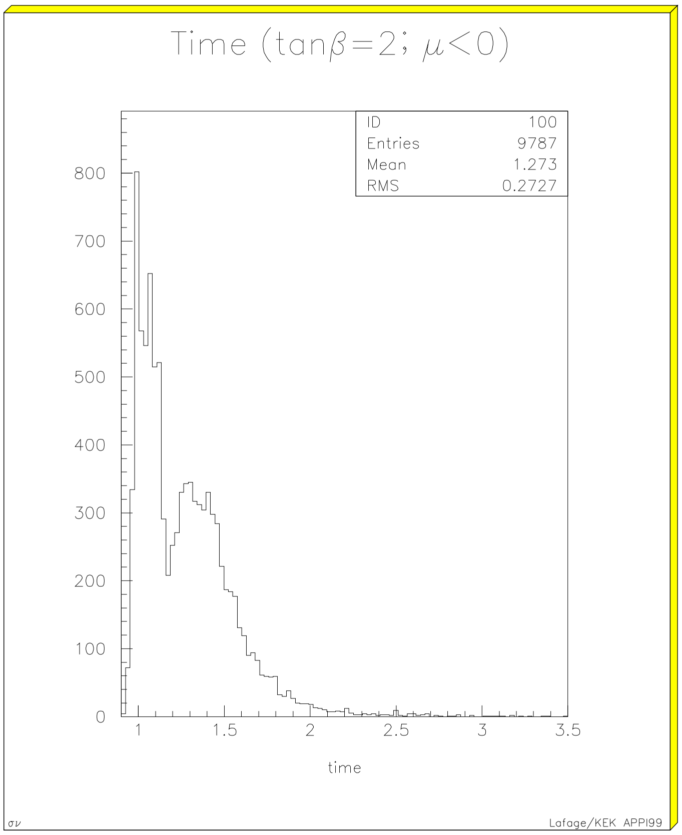
α_s (tanβ=2; μ>0)
α_{sDR}
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.08 0.09 0.1 0.11 0.12 0.13 0.14 0.15 0.16
α_{sMS}
Lafage/KEK APPI99

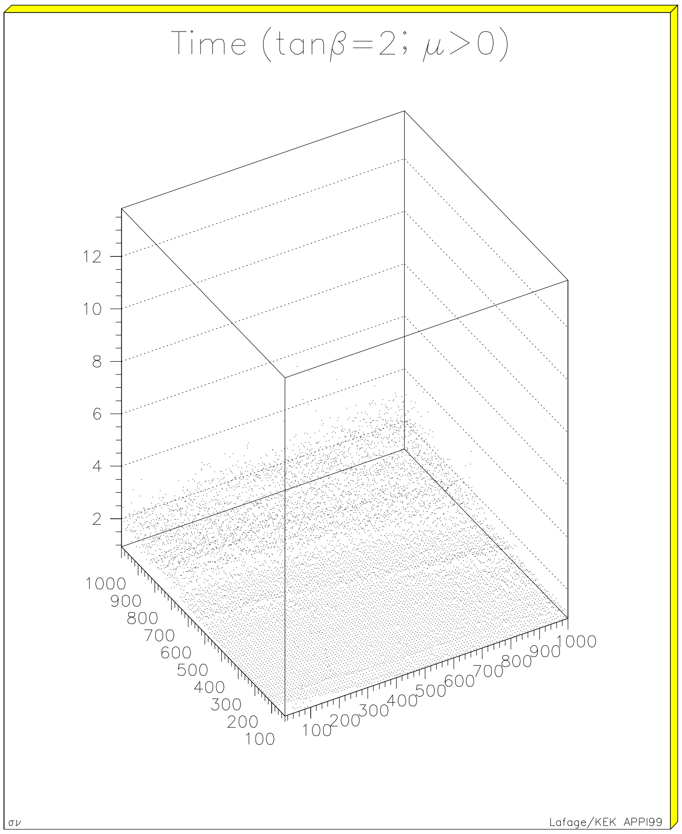


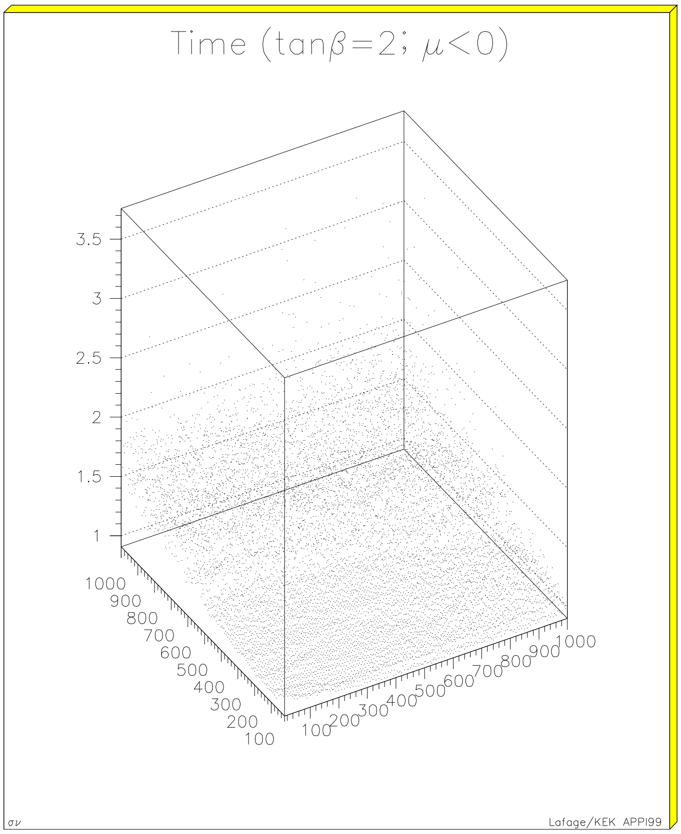
α_s (tanβ=2; μ<0)
α_{sDR}
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.1 0.11 0.12 0.13 0.14 0.15
α_{sMS}
Lafage/KEK APPI99

Efficiency


Time (tanβ=2; μ>0)
ID 100
Entries 9843
Mean 1.445
RMS 0.4440
600
500
400
300
200
100
0
1 1.5 2 2.5 3 3.5
σν
Lafage/KEK APPI99


Time (tanβ=2; μ<0)
ID 100
Entries 9787
Mean 1.273
RMS 0.2727
800
700
600
500
400
300
200
100
0
1 1.5 2 2.5 3 3.5
time
σν
Lafage/KEK APPI99


Time (tanβ=2; μ>0)
12
10
8
6
4
2
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
σν
Lafage/KEK APPI99


Time (tanβ=2; μ<0)
3.5
3
2.5
2
1.5
1
1000 900 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 900 1000
σν
Lafage/KEK APPI99

Projects

- **comparison with other integrators (`Isajet`, `Spythia`, `SuSpect`) is well advanced.**
- **comparison of self-energies is under way.**
- **more refined thresholds**
- **3-loop Yukawa's coupling β function**
- **others self-energies (-inos...)**
- **cleaner exception management (`NaN`...)**
- **alternative expression of symmetry breaking**
- **more scales ($\Lambda_{EWSB} \neq M_Z$)**