Slepton NLG (Non-linear gauge) in GRACE/SUSY-loop

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Contents

1. Motivation

- SUSY is an attractive candidate for BSM.

  ➡ Search for SUSY particles at LHC and ILC

- Experiments at the ILC offer high-precision determination of SUSY parameters.

  ➡ Theoretical predictions with the similarly high accuracy is required.

- Contributions at least one-loop level should be included in perturbative calculations of amplitudes.

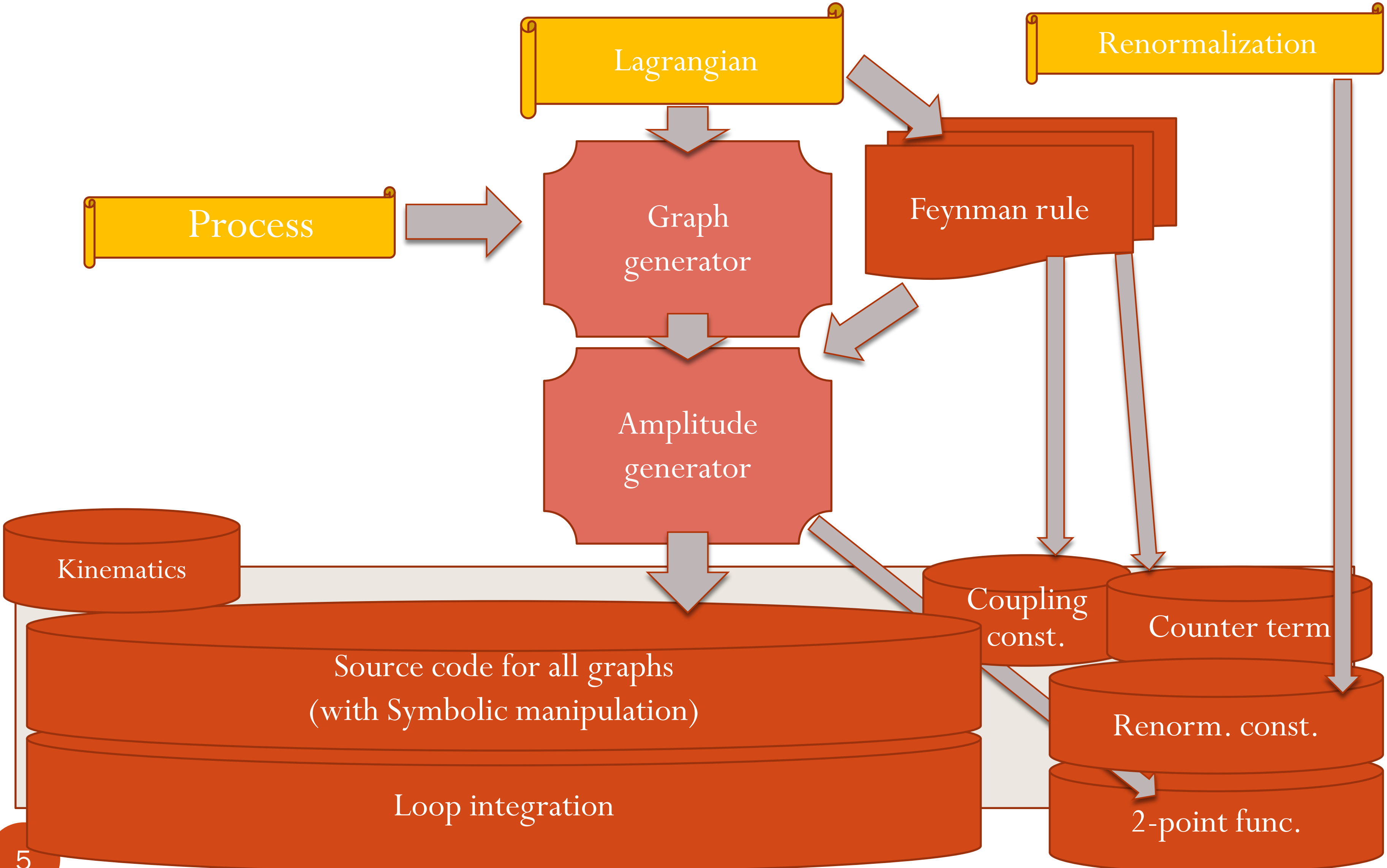
  ➡ Automatic calculation

2. GRACE system

- Generates all Feynman diagrams automatically
- Generates physical amplitudes automatically
- Incorporates libraries (2-point functions, Loop integral, Kinematics, etc.)
- Integrates the matrix element by the adaptive Monte Carlo method (**BASES**)
- Does Monte Carlo event generation (**SPRING**)
- Enables various self-check for the results (UV cancellation, IR cancellation, NLG invariance, etc.)

Other automatic SUSY 1-loop systems
SloopS (F. Boudjema, et al., 2005), **FeynArt/Calc** (T. Hahn, 2001, 2006)

Structure of automatic systems for loop calculation



Features of **GRACE**

	SM	MSSM
Tree level	**GRACE**	**GRACE/SUSY**
1-loop level	**GRACE-loop**	**GRACE/SUSY-loop**

http://minami-home.kek.jp/

Features of **GRACE** Calculation Methods

	SM	MSSM
Tree level	**GRACE**	**GRACE/SUSY**
	Helicity amplitudes (**CHANEL**)	
1-loop level	**GRACE-loop**	**GRACE/SUSY-loop**
	Matrix elements (**REDUCE, FORM**)	(**REDUCE**)

Features of **GRACE** [Gauge-invariance tests]

	SM	MSSM
Tree level	**GRACE**	**GRACE/SUSY**
	Between covariant and unitary gauge	
1-loop level	**GRACE-loop**	**GRACE/SUSY-loop**
	Non-linear gauge (NLG)	

Remarks:

- In general R_ξ-gauge, the longitudinal $k_\mu k_\nu$ part of the gauge propagators, e.g.

$$\frac{1}{k^2 - M_W^2 + i\varepsilon}\left(g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right)$$

  causes numerical instability in loop calculation.
- In unitary gauge ($\xi_W \to \infty$), situation is worse.
- We need another gauge fixing along with 't Hooft-Feynman gauge ($\xi_W = 1$) for gauge-invariance test.

➡ Non-linear gauge

3. Non-linear gauge (NLG)

The NLG formalism is an extension of the linear R_ξ-gauge:

$$L_{\mathrm{gf}} = -\frac{1}{\xi_W}\left|F_{W^+}\right|^2 - \frac{1}{2\xi_Z}\left(F_Z\right)^2 - \frac{1}{2\xi_\gamma}\left(F_\gamma\right)^2$$

(gauge fixing terms)

$$F_{W^+} = \partial_\mu W^{+\mu} + i\xi_W \frac{g}{2} v G^+$$

$$F_{W^-} = \partial_\mu W^{-\mu} - i\xi_W \frac{g}{2} v G^-$$

$$\left(M_W = \frac{g}{2} v, \quad M_Z = \frac{g_Z}{2} v\right)$$

$$F_Z = \partial_\mu Z^\mu + \xi_Z \frac{g_Z}{2} v G^0$$

$$F_\gamma = \partial_\mu A^\mu$$

NLG formalism:

$$L_{\rm gf} = -\frac{1}{\xi_W}\left|F_{W^+}\right|^2 - \frac{1}{2\xi_Z}\left(F_Z\right)^2 - \frac{1}{2\xi_\gamma}\left(F_\gamma\right)^2$$

(gauge fixing terms)

$$F_{W^+} = (\partial_\mu \underline{+ie\tilde{\alpha}A_\mu + igc_W\tilde{\beta}Z_\mu})W^{+\mu} + i\xi_W\frac{g}{2}(v \underline{+ \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 + i\tilde{\kappa}G^0})G^+$$

$$F_{W^-} = (\partial_\mu \underline{-ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu})W^{-\mu} - i\xi_W\frac{g}{2}(v \underline{+ \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 - i\tilde{\kappa}G^0})G^-$$

$$F_Z = \partial_\mu Z^\mu + \xi_Z\frac{g_Z}{2}(v \underline{+ \tilde{\varepsilon}_h h^0 + \tilde{\varepsilon}_H H^0})G^0 \qquad F_\gamma = \partial_\mu A^\mu$$

NLG parameters: $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_h, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\varepsilon}_h, \tilde{\varepsilon}_H)$

SM: G. Bélanger et al., Phys. Rep. 430 (2006) 117

MSSM: J. Fujimoto et al., Nucl. Phys. (Proc. Suppl.) 157 (2006) 157; Phys. Rev. D75 (2007) 113002

Extension of NLG by including bilinear forms of sleptons

$$L_{\rm gf} = -\frac{1}{\xi_W}\left|F_{W^+}\right|^2 - \frac{1}{2\xi_Z}\left(F_Z\right)^2 - \frac{1}{2\xi_\gamma}\left(F_\gamma\right)^2$$

(gauge fixing terms)

$$F_{W^+} = (\partial_\mu + ie\tilde{\alpha}A_\mu + igc_W\tilde{\beta}Z_\mu)W^{+\mu} + i\xi_W\frac{g}{2}(v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 + i\tilde{\kappa}G^0)G^+$$

$$\boxed{+i\xi_W g\sum_{i=1,2}\left\{\tilde{c}_i^e\left(\tilde{e}_i^*\tilde{\nu}_e\right) + \tilde{c}_i^\mu\left(\tilde{\mu}_i^*\tilde{\nu}_\mu\right) + \tilde{c}_i^\tau\left(\tilde{\tau}_i^*\tilde{\nu}_\tau\right)\right\}}$$

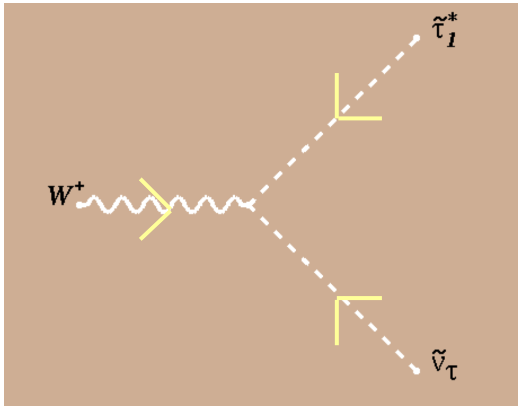
$$F_{W^-} = (\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{-\mu} - i\xi_W\frac{g}{2}(v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 - i\tilde{\kappa}G^0)G^-$$

$$\boxed{-i\xi_W g\sum_{i=1,2}\left\{\tilde{c}_i^e\left(\tilde{\nu}_e^*\tilde{e}_i\right) + \tilde{c}_i^\mu\left(\tilde{\nu}_\mu^*\tilde{\mu}_i\right) + \tilde{c}_i^\tau\left(\tilde{\nu}_\tau^*\tilde{\tau}_i\right)\right\}}$$

$$F_Z = \partial_\mu Z^\mu + \xi_Z\frac{g_Z}{2}(v + \tilde{\varepsilon}_h h^0 + \tilde{\varepsilon}_H H^0)G^0 \qquad F_\gamma = \partial_\mu A^\mu$$

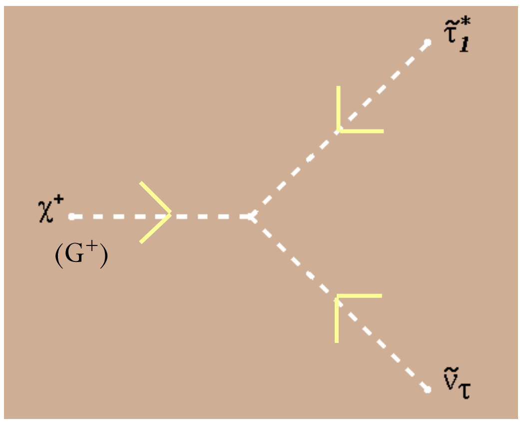
NLG parameters: $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_h, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\varepsilon}_h, \tilde{\varepsilon}_H, \tilde{c}_i^e, \tilde{c}_i^\mu, \tilde{c}_i^\tau)$

Feynman rules ($\xi_W = 1$)



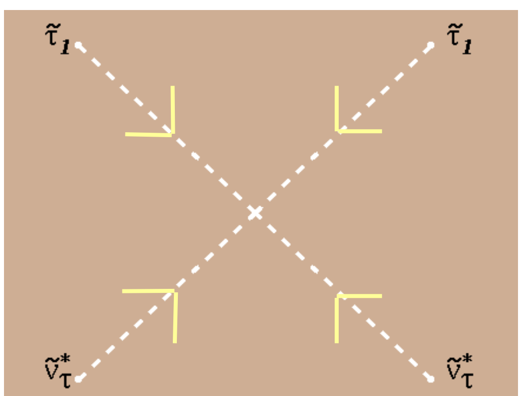
$$i\frac{g}{\sqrt{2}}\cos\theta_\tau\left(p_\mu^{\tilde{\nu}} - p_\mu^{\tilde{\tau}}\right) \boxed{-ig\tilde{c}_1^{\tau}\left(p_\mu^{\tilde{\nu}} + p_\mu^{\tilde{\tau}}\right)}$$

Linear gauge + NLG



$$i\frac{g}{\sqrt{2}M_W}\cos\theta_\tau\left(M_W^2\cos 2\beta - m_\tau^2\right) \boxed{-igM_W\tilde{c}_1^{\tau}}$$

Linear gauge + NLG



$$-i\frac{g_Z^2}{4}\cos^2\theta_\tau + i\frac{1}{2}\sin^2\theta_\tau\left(g_Z^2\sin^2\theta_W - \frac{g^2 m_\tau^2}{M_W^2\cos^2\beta}\right)$$

Linear gauge +

$$\boxed{-ig^2\left(\tilde{c}_1^{\tau}\right)^2}$$

NLG

4. Renormalization scheme

- The renormalization scheme for the electroweak sector in GRACE/SUSY-loop is a variation of the on-mass-shell scheme [1].
- The on-mass-shell condition:

gauge bosons (W, Z, γ) all fermions (f)

all sfermions ($\tilde{f}$)

CP odd Higgs (A^0) the heavier CP even Higgs (H^0)

both charginos ($\tilde{\chi}_1^+, \tilde{\chi}_2^+$)

the lightest neutralino ($\tilde{\chi}_1^0$)

[1] J. Fujimoto, T. Ishikawa, Y. Kurihara, M. Jimbo, T. Kon and M. Kuroda, Phys. Rev. D75 113002 (2007).

Renormalization conditions in slepton sector

RC-I (used in [1])

Wavefunction renormalization constants

intrduced in mass eigenstates

$$\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}_0 = \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix}_0 \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}_0 \qquad \tilde{f} = \tilde{e}, \tilde{\mu}, \tilde{\tau}$$

$$= \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix}_0 \begin{pmatrix} Z_{11}^{1/2} & Z_{12}^{1/2} \\ Z_{21}^{1/2} & Z_{22}^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} = \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

Renormalization conditions in "RC-I"

(The third generation of slepton sector)

2-point func. (diagonal)

- On mass-shell conditions: $\delta m_{\tilde{f}}^2 = -\Sigma_{\tilde{f}\tilde{f}}(m_{\tilde{f}}^2), \quad \tilde{f} = \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_\tau$
- Residue conditions:

$$\delta Z_{\tilde{\tau}_i \tilde{\tau}_i} = \frac{\partial}{\partial q^2} \Sigma_{\tilde{\tau}_i \tilde{\tau}_i}(q^2)\Big|_{q^2 \to m_{\tilde{\tau}_i}^2} \equiv \Sigma'_{\tilde{\tau}_i \tilde{\tau}_i}(m_{\tilde{\tau}_i}^2)$$

$$\delta Z_{\tilde{\nu}_\tau} = \Sigma'_{\tilde{\nu}_\tau \tilde{\nu}_\tau}(m_{\tilde{\nu}_\tau}^2)$$

- Decoupling conditions:

$$\frac{1}{2}\delta Z_{\tilde{\tau}_i \tilde{\tau}_j} = -\frac{\Sigma_{\tilde{\tau}_i \tilde{\tau}_j}(m_{\tilde{f}_j}^2)}{m_{\tilde{\tau}_i}^2 - m_{\tilde{\tau}_j}^2}, \quad i \neq j$$

- SU(2) relation:

$$\delta\theta_\tau = [\delta m_{\tilde{\nu}_\tau}^2 - \delta(M_W^2 \cos 2\beta - m_\tau^2) - \cos^2\theta_\tau \delta m_{\tilde{\tau}_1}^2 - \sin^2\theta_\tau \delta m_{\tilde{\tau}_2}^2] \times \frac{1}{\sin 2\theta_\tau (m_{\tilde{\tau}_2}^2 - m_{\tilde{\tau}_1}^2)}$$

RC-II

Wavefunction renormalization constants

➡ intrduced in gauge symmetric states

(The third generation of slepton sector)

$$\begin{pmatrix} \tilde{\tau}_{\mathrm{L}} \\ \tilde{\tau}_{\mathrm{R}} \end{pmatrix}_0 = \begin{pmatrix} Z_{\mathrm{LL}}^{1/2} & Z_{\mathrm{LR}}^{1/2} \\ Z_{\mathrm{RL}}^{1/2} & Z_{\mathrm{RR}}^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_{\mathrm{L}} \\ \tilde{\tau}_{\mathrm{R}} \end{pmatrix}$$

$$\tilde{\nu}_{\tau 0} = Z_{\tilde{\nu}_\tau}^{1/2} \, \tilde{\nu}_\tau$$

$$\left(m_{\tilde{\nu}_\tau}^2\right)_0 = m_{\mathrm{L}0}^2 + (M_W \cos 2\beta - m_\tau^2)_0$$

$$\left(\tilde{\tau}_L^* \ \tilde{\tau}_R^*\right)_0 \begin{pmatrix} Z_{LL}^{1/2} & Z_{RL}^{1/2} \\ Z_{LR}^{1/2} & Z_{RR}^{1/2} \end{pmatrix} \begin{pmatrix} m_L^2 & m_{LR}^2 \\ m_{LR}^2 & m_R^2 \end{pmatrix}_0 \begin{pmatrix} Z_{LL}^{1/2} & Z_{LR}^{1/2} \\ Z_{RL}^{1/2} & Z_{RR}^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}_0$$

$$= \left(\tilde{\tau}_L^* \ \tilde{\tau}_R^*\right)_0 \left(M + \delta M + \frac{1}{2} \begin{pmatrix} \delta Z_{LL} & \delta Z_{RL} \\ \delta Z_{LR} & \delta Z_{RR} \end{pmatrix} M + \frac{1}{2} M \begin{pmatrix} \delta Z_{LL} & \delta Z_{LR} \\ \delta Z_{RL} & \delta Z_{RR} \end{pmatrix} \right) \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}_0$$

$$\equiv \left(\tilde{\tau}_1^* \ \tilde{\tau}_2^*\right) \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} + \left(\tilde{\tau}_1^* \ \tilde{\tau}_2^*\right) \begin{pmatrix} \delta m_{11}^2 & \delta m_{12}^2 \\ \delta m_{12}^2 & \delta m_{22}^2 \end{pmatrix} \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix}$$

$$\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = \begin{pmatrix} \cos\theta_\tau & \sin\theta_\tau \\ -\sin\theta_\tau & \cos\theta_\tau \end{pmatrix} M \begin{pmatrix} \cos\theta_\tau & -\sin\theta_\tau \\ \sin\theta_\tau & \cos\theta_\tau \end{pmatrix} \qquad M \equiv \begin{pmatrix} m_L^2 & m_{LR}^2 \\ m_{LR}^2 & m_R^2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_\tau & \sin\theta_\tau \\ -\sin\theta_\tau & \cos\theta_\tau \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

Renormalization conditions in "RC-II"

- On mass-shell conditions:

$$0=\Sigma_{11}(m_1^2)+\delta m_{11}^2-m_1^2[\cos^2\theta_\tau\delta Z_{\mathrm{LL}}+\cos\theta_\tau\sin\theta_\tau(\delta Z_{\mathrm{LR}}+\delta Z_{\mathrm{RL}})+\sin^2\theta_\tau\delta Z_{\mathrm{RR}}]$$

$$0=\Sigma_{22}(m_2^2)+\delta m_{22}^2-m_2^2[\sin^2\theta_\tau\delta Z_{\mathrm{LL}}-\cos\theta_\tau\sin\theta_\tau(\delta Z_{\mathrm{LR}}+\delta Z_{\mathrm{RL}})+\cos^2\theta_\tau\delta Z_{\mathrm{RR}}]$$

$$0=\Sigma_{\tilde{\nu}\tilde{\nu}}(m_{\tilde{\nu}}^2)+\delta m_{\tilde{\nu}}^2$$

- Residue conditions:

$$0=\Sigma_{11}'(m_1^2)-[\cos^2\theta_\tau\delta Z_{\mathrm{LL}}+\cos\theta_\tau\sin\theta_\tau(\delta Z_{\mathrm{LR}}+\delta Z_{\mathrm{RL}})+\sin^2\theta_\tau\delta Z_{\mathrm{RR}}]$$

$$0=\Sigma_{22}'(m_2^2)-[\sin^2\theta_\tau\delta Z_{\mathrm{LL}}-\cos\theta_\tau\sin\theta_\tau(\delta Z_{\mathrm{LR}}+\delta Z_{\mathrm{RL}})+\cos^2\theta_\tau\delta Z_{\mathrm{RR}}]$$

$$0=\Sigma_{\tilde{\nu}\tilde{\nu}}'(m_{\tilde{\nu}}^2)-\delta Z_{\tilde{\nu}}$$

- Decoupling conditions:

$$0=\Sigma_{12}(m_1^2)+\delta m_{12}^2-\frac{m_1^2}{2}[\sin 2\theta_\tau(\delta Z_{\rm LL}-\delta Z_{\rm RR})+\cos 2\theta_\tau(\delta Z_{\rm LR}+\delta Z_{\rm RL})]$$

$$0=\Sigma_{12}(m_2^2)+\delta m_{12}^2-\frac{m_2^2}{2}[\sin 2\theta_\tau(\delta Z_{\rm RR}-\delta Z_{\rm LL})+\cos 2\theta_\tau(\delta Z_{\rm LR}+\delta Z_{\rm RL})]$$

- SU(2) relation:

$$\begin{aligned} m_{\tilde{\nu}_\tau}^2 &= m_{\rm L}^2+M_W\cos 2\beta-m_\tau^2 \\ &= \cos^2\theta\, m_1^2+\sin^2\theta\, m_2^2+M_W\cos 2\beta-m_\tau^2 \end{aligned}$$

$$\begin{aligned} \delta m_{\tilde{\nu}_\tau}^2 &= \delta m_{\rm L}^2+\delta(M_W^2\cos 2\beta-m_\tau^2) \\ &= \cos^2\theta_\tau\delta m_1^2+\sin^2\theta_\tau\delta m_2^2-\cos 2\theta_\tau\delta m_{12}^2-m_{\rm LL}^2\delta Z_{\rm LL}-m_{\rm LR}^2\delta Z_{\rm RL} \\ &\quad +\delta(M_W^2\cos 2\beta-m_\tau^2) \end{aligned}$$

5. Numerical tests

- Four kinds of decay processes:

  1) $\tilde{\tau}_2^- \to \tilde{\tau}_1^- + Z^0$ 2) $\tilde{\tau}_2^- \to \tilde{\tau}_1^- + h^0$

  3) $\tilde{\nu}_\tau \to \tilde{\tau}_1^- + W^+$ 4) $\tilde{\tau}_1^- \to \nu_\tau + \tilde{\chi}_1^-$

- Two kinds of scattering processes:

  1) $\tilde{\tau}_1 + \tilde{\tau}_1^* \to \tilde{\tau}_1 + \tilde{\tau}_1^*$ 2) $\tilde{\nu}_\tau + \tilde{\nu}_\tau^* \to \tilde{\nu}_\tau + \tilde{\nu}_\tau^*$

- Two kinds of renormalization conditions:

  1) "RC-I" 2) "RC-II"

- NLG input-parameter:

$$F_{W^+} = \partial_\mu W^{+\mu} + i\xi_W \frac{g}{2} v G^+ + i\xi_W g \tilde{c}_1^\tau \left(\tilde{\tau}_1^* \tilde{\nu}_\tau \right)$$

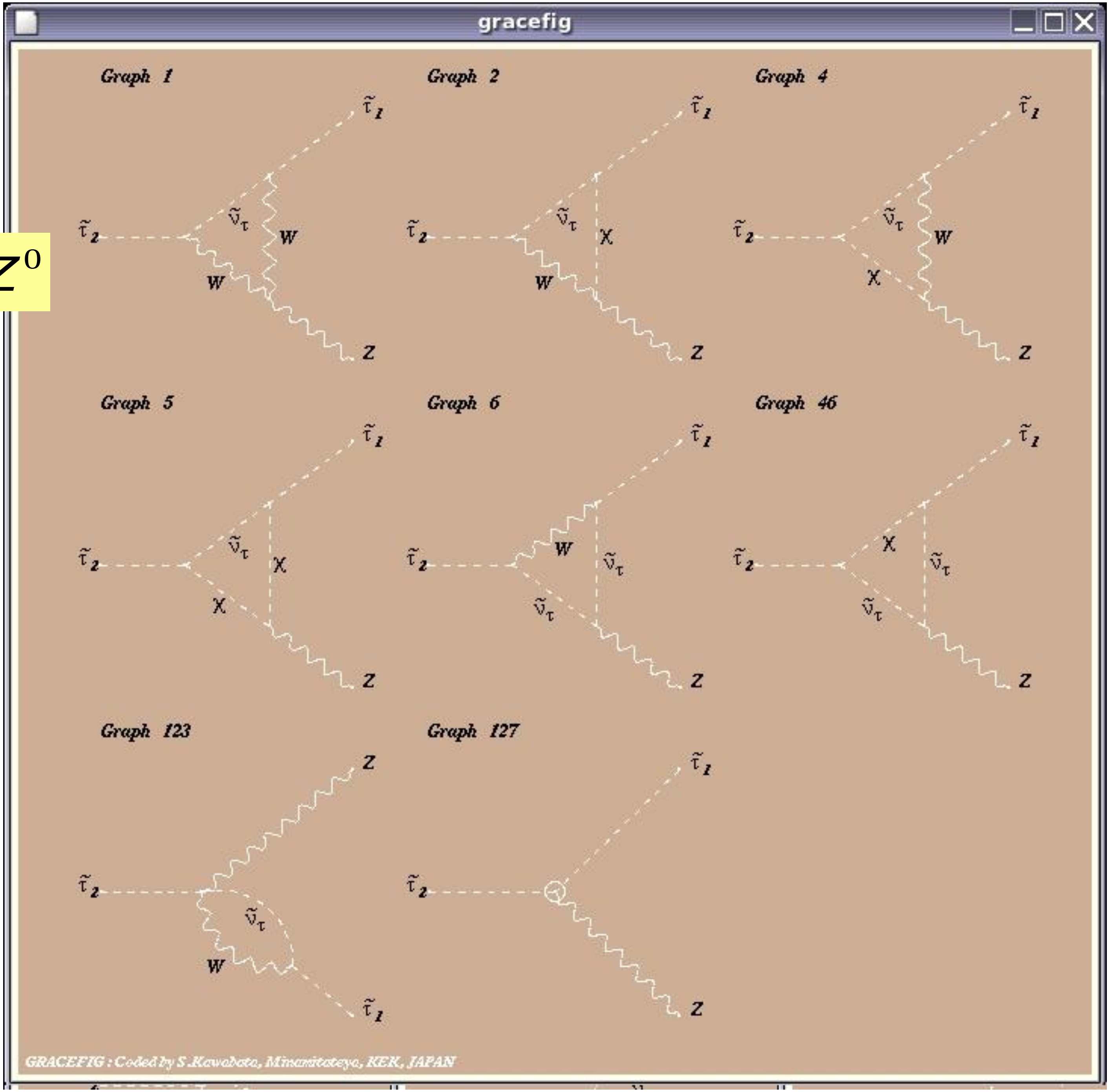
- Kinematically allowed settings for SUSY parameters

Decay

Process 1

$$\tilde{\tau}_2^{\,-} \rightarrow \tilde{\tau}_1^{\,-} + Z^0$$

Loop + CT
134 diagrams



CUV part

Process 1

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ in **ANS**(CUV=1000) - **ANS**(CUV=0)

Linear gauge

RC-I

Graph no.	n=0	n=1
1	6.272271E+04	6.385803E+04
6	-2.692742E+04	-2.741483E+04
123	9.029780E+03	-6.128816E+03
127	-1.008044E+05	-3.031439E+04
sum	2.414190E-20	1.662919E-27
max	1.008044E+05	6.385803E+04

RC-II

Graph no.	n=0	n=1
1	6.272271E+04	6.385803E+04
6	-2.692742E+04	-2.741483E+04
123	9.029780E+03	-6.128816E+03
127	-1.008044E+05	-3.031439E+04
sum	2.414189E-20	1.640723E-27
max	1.008044E+05	6.385803E+04

Finite part

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ in **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	-7.286927E+02	-6.842374E+02
2	-2.348766E+01	-5.343054E+00
4	2.694918E+01	-3.976518E+00
5	-4.323101E+01	-9.834338E+00
6	4.214998E+02	2.745840E+02
46	7.246966E+01	1.648565E+01
123	-9.946234E+01	7.238315E+01
127	-2.103937E+04	3.399385E+02
sum	1.210550E+03	-1.982592E-24
max	2.125460E+04	6.842374E+02

RC-II

Graph no.	n=0	n=1
1	-7.286927E+02	-6.842374E+02
2	-2.348766E+01	-5.343054E+00
4	2.694918E+01	-3.976518E+00
5	-4.323101E+01	-9.834338E+00
6	4.214998E+02	2.745840E+02
46	7.246966E+01	1.648565E+01
123	-9.946234E+01	7.238315E+01
127	-2.103937E+04	3.399385E+02
sum	1.210550E+03	-1.982592E-24
max	2.125460E+04	6.842374E+02

Process 2 $\tilde{\tau}_2^- \to \tilde{\tau}_1^- + h^0$

Loop + CT
151 diagrams

CUV part Process 2

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^{n}$ in **ANS**(CUV=1000) - **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	-2.864048E+02	-5.831779E+02
3	1.281803E+03	2.915889E+02
4	-2.459420E+02	-5.007874E+02
7	-1.470710E+03	-2.994661E+03
10	1.839419E+02	3.745426E+02
135	0.000000E+00	2.915889E+02
138	-1.279135E+03	-2.909820E+02
145	5.330665E+02	3.411888E+03
sum	-6.032040E-20	-5.762234E-27
max	2.366182E+03	3.411888E+03

RC-II

Graph no.	n=0	n=1
1	-2.864048E+02	-5.831779E+02
3	1.281803E+03	2.915889E+02
4	-2.459420E+02	-5.007874E+02
7	-1.470710E+03	-2.994661E+03
10	1.839419E+02	3.745426E+02
135	0.000000E+00	2.915889E+02
138	-1.279135E+03	-2.909820E+02
145	5.330665E+02	3.411888E+03
sum	-6.032040E-20	-5.755505E-27
max	2.366182E+03	3.411888E+03

Finite part

Process 2

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^{n}$ in **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	1.431358E+01	6.114628E+00
3	-1.902592E+01	-4.328082E+00
4	2.291272E+00	5.997467E+00
6	-1.678165E+00	-3.817546E-01
7	3.674159E+00	3.338447E+01
9	3.940194E+00	8.963287E-01
10	-3.333833E+00	-4.016341E+00
50	-8.074984E-01	-1.836925E-01
135	0.000000E+00	-2.485908E+00
138	1.434392E+01	3.263005E+00
145	-1.131787E+04	-3.826012E+01
sum	-1.117318E+03	-3.960740E-22
max	1.849677E+04	3.826012E+01

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^{n}$ in **ANS**(CUV=0)

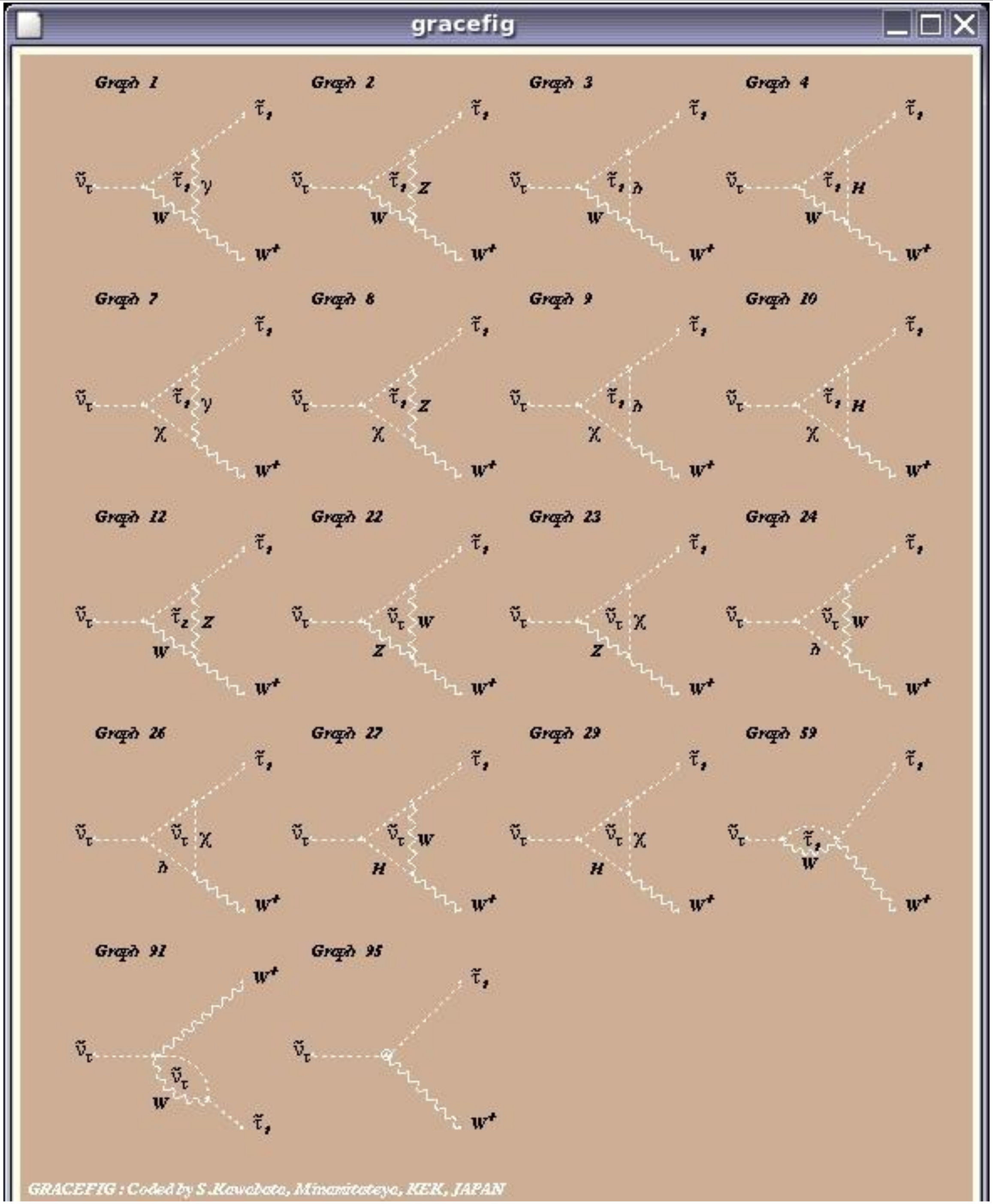
RC-II

Graph no.	n=0	n=1
1	1.431358E+01	6.114628E+00
3	-1.902592E+01	-4.328082E+00
4	2.291272E+00	5.997467E+00
6	-1.678165E+00	-3.817546E-01
7	3.674159E+00	3.338447E+01
9	3.940194E+00	8.963287E-01
10	-3.333833E+00	-4.016341E+00
50	-8.074984E-01	-1.836925E-01
135	0.000000E+00	-2.485908E+00
138	1.434392E+01	3.263005E+00
145	-1.131787E+04	-3.826012E+01
sum	-1.117318E+03	-3.960740E-22
max	1.849677E+04	3.826012E+01

Process 3

$$\tilde{\nu}_\tau \rightarrow \tilde{\tau}_1^- + W^+$$

Loop + CT
101 diagrams



CUV part

Process 3

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ in **ANS**(CUV=1000) - **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	8.037124E+04	-8.182600E+04
2	6.338572E+03	-6.453304E+03
22	1.797544E+05	1.830081E+05
59	-4.335490E+04	-2.942643E+04
91	-8.987721E+04	6.100269E+04
95	-5.321610E+05	-1.263050E+05
sum	8.513597E-19	-9.750321E-26
max	5.321610E+05	1.830081E+05

RC-II

Graph no.	n=0	n=1
1	8.037124E+04	-8.182600E+04
2	6.338572E+03	-6.453304E+03
22	1.797544E+05	1.830081E+05
59	-4.335490E+04	-2.942643E+04
91	-8.987721E+04	6.100269E+04
95	-5.321610E+05	-1.263050E+05
sum	8.513598E-19	-9.749545E-26
max	5.321610E+05	1.830081E+05

Finite part

Process 3

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ in **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	-6.731073E+03	1.600386E+03
2	-6.186723E+01	6.182872E+01
3	-6.368230E+02	-1.180042E+02
4	3.836433E+01	1.075723E+01
7	-7.063331E+03	-8.245146E+02
8	-5.471407E+00	-6.386866E-01
9	1.010901E+03	1.180042E+02
10	-9.215341E+01	-1.075723E+01
22	-2.065451E+03	-2.057789E+03
23	7.232945E+01	8.443140E+00
24	-1.852899E+01	2.957657E+00
26	-2.533722E+01	-2.957657E+00
27	-3.350109E-03	8.422777E-04
29	-7.215500E-03	-8.422777E-04
59	4.110899E+02	3.165356E+02
91	1.006455E+03	-7.279751E+02
95	-1.773689E+05	1.623724E+03
sum	9.335785E+04	9.752294E-28
max	1.941086E+05	2.057789E+03

Finite part

Process 3

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^{\mathrm{n}}$ in **ANS**(CUV=0)

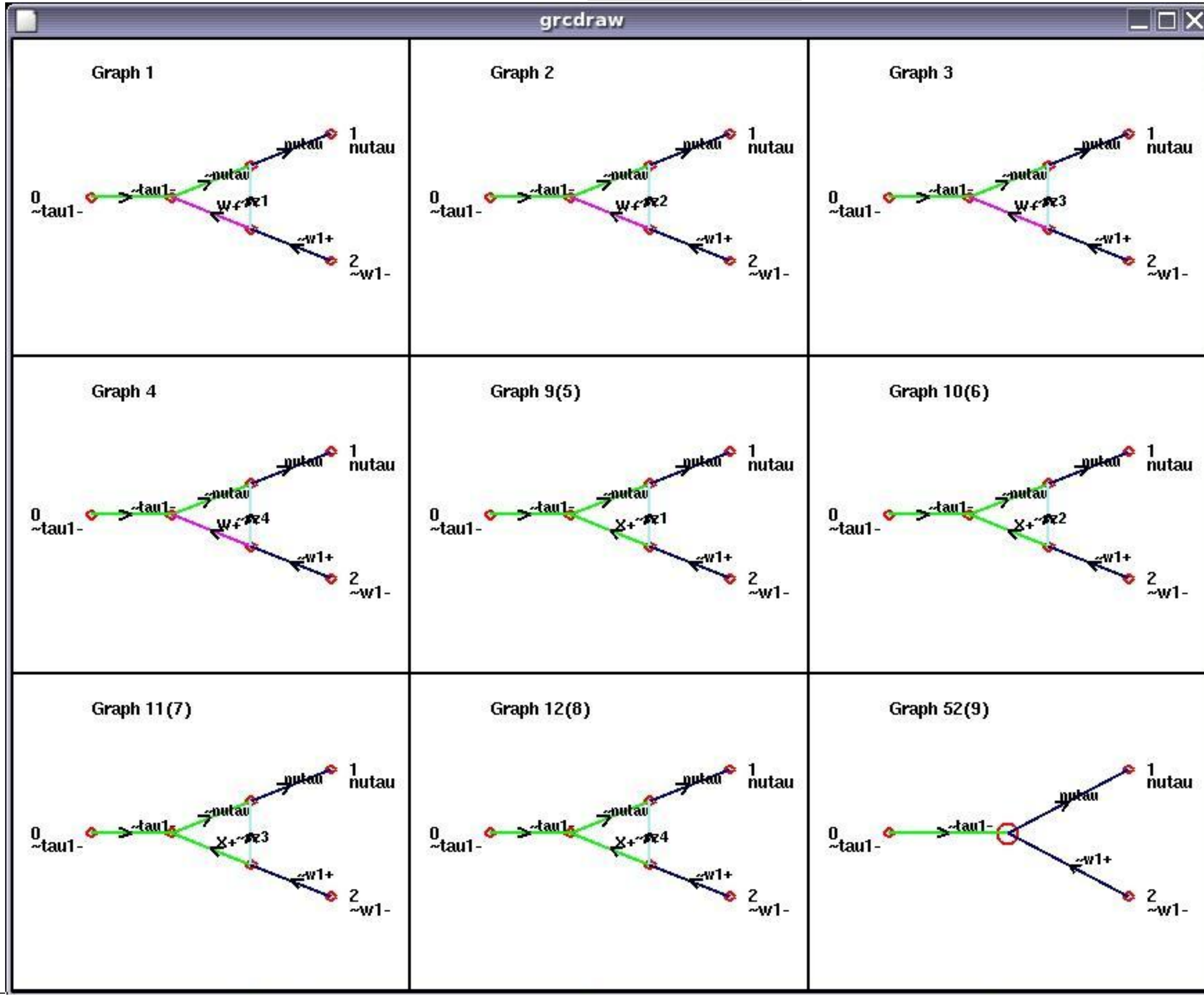
RC-II

Graph no.	n=0	n=1
1	-6.731073E+03	1.600386E+03
2	-6.186723E+01	6.182872E+01
3	-6.368230E+02	-1.180042E+02
4	3.836433E+01	1.075723E+01
7	-7.063331E+03	-8.245146E+02
8	-5.471407E+00	-6.386866E-01
9	1.010901E+03	1.180042E+02
10	-9.215341E+01	-1.075723E+01
22	-2.065451E+03	-2.057789E+03
23	7.232945E+01	8.443140E+00
24	-1.852899E+01	2.957657E+00
26	-2.533722E+01	-2.957657E+00
27	-3.350109E-03	8.422777E-04
29	-7.215500E-03	-8.422777E-04
59	4.110899E+02	3.165356E+02
91	1.006455E+03	-7.279751E+02
95	-1.773689E+05	1.623724E+03
sum	9.335785E+04	9.761524E-28
max	1.941086E+05	2.057789E+03

Process 4

$$\tilde{\tau}_1^- \to \nu_\tau + \tilde{\chi}_1^-$$

Loop + CT
56 diagrams



CUV part

Process 4

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^{\mathrm{n}}$ in **ANS**(CUV=1000) - **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	4.853635E+03	9.882976E+03
2	-2.692561E+03	-5.482596E+03
3	-1.126952E+03	-2.294701E+03
4	9.578295E+02	1.950333E+03
52	-1.351460E+05	-4.056013E+03
sum	-2.665045E-18	4.244246E-26
max	1.351460E+05	9.882976E+03

RC-II

Graph no.	n=0	n=1
1	4.853635E+03	9.882976E+03
2	-2.692561E+03	-5.482596E+03
3	-1.126952E+03	-2.294701E+03
4	9.578295E+02	1.950333E+03
52	-1.351460E+05	-4.056013E+03
sum	-2.665045E-18	4.253882E-26
max	1.351460E+05	9.882976E+03

Finite part

Process 4

Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ in **ANS**(CUV=0)

RC-I

Graph no.	n=0	n=1
1	-7.603322E+01	-1.069756E+02
2	4.143377E+01	5.958872E+01
3	1.687706E+01	2.528120E+01
4	-1.476331E+01	-2.204968E+01
9	3.167550E-05	-2.141832E-01
10	1.848748E-05	-1.250085E-01
11	5.813461E-05	-3.930944E-01
12	-1.325900E-04	8.965466E-01
52	5.832838E+03	4.399113E+01
sum	1.461290E+03	7.694140E-24
max	5.832838E+03	1.069756E+02

RC-II

Graph no.	n=0	n=1
1	-7.603322E+01	-1.069756E+02
2	4.143377E+01	5.958872E+01
3	1.687706E+01	2.528120E+01
4	-1.476331E+01	-2.204968E+01
9	3.167550E-05	-2.141832E-01
10	1.848748E-05	-1.250085E-01
11	5.813461E-05	-3.930944E-01
12	-1.325900E-04	8.965466E-01
52	5.832838E+03	4.399113E+01
sum	1.461290E+03	7.694139E-24
max	5.832838E+03	1.069756E+02

Scattering

Process 1 $\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$

- [Loop+CT: 3288 diagrams] X [Tree: 11 diagrams]
- Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ $(n = 0,1,2,3,4)$ in **ANS**(CUV=1000) – **ANS**(CUV=0) canceled among 149 diagrams.

	n=0	n=1	n=2	n=3	n=4
sum	-1.108258E-21	-1.813692E-25	-9.099667E-29	2.117843E-31	4.801768E-38
max	1.777580E+02	1.914021E+02	3.376633E-01	3.266330E-01	1.662726E-01

- Coefficients of $\left(\tilde{c}_1^{\tau}\right)^n$ $(n = 1,2,3,4)$ in **ANS**(CUV=0) canceled among 243 diagrams.

	n=0	n=1	n=2	n=3	n=4
sum	1.286445E+04	3.341261E-27	-3.304929E-29	-4.964405E-29	-2.529046E-29
max	7.350494E+03	2.146339E+00	9.943723E-02	3.865195E-03	2.111032E-03

CUV part

$$\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$$

Graph no.	n=0	n=1	n=2	n=3	n=4
21	-1.338260E-05	-2.724966E-05	0	0	0
28	4.569066E-07	9.303537E-07	0	0	0
93	-1.064284E-06	-2.167096E-06	0	0	0
●●●	●●●	●●●	●●●	●●●	●●●
798	-1.767730E-03	-7.198907E-03	-7.329211E-03	0	0
931	-9.672486E-03	-7.878050E-02	-2.406194E-01	-3.266330E-01	-1.662726E-01
940	-9.672486E-03	-7.878050E-02	-2.406194E-01	-3.266330E-01	-1.662726E-01
●●●	●●●	●●●	●●●	●●●	●●●
3102	0	0	4.010324E-02	0	0
3125	-6.313793E-03	0	-6.480157E-02	0	-1.662726E-01
3160	0	0	4.010324E-02	0	0
●●●	●●●	●●●	●●●	●●●	●●●
3181	7.442611E+01	-2.060637E-02	0	0	0
3182	7.350494E+03	-2.146339E+00	0	0	0
sum	-1.108258E-21	-1.813692E-25	-9.099667E-29	2.117843E-31	4.801768E-38
max	1.777580E+02	1.914021E+02	3.376633E-01	3.266330E-01	1.662726E-01

Process 2

$$\widetilde{\nu}_\tau + \widetilde{\nu}_\tau^* \rightarrow \widetilde{\nu}_\tau + \widetilde{\nu}_\tau^*$$

- [Loop+CT: 2470 diagrams] ╳ [Tree: 9 diagrams]
- Coefficients of $\left(\tilde{c}_1^\tau\right)^n$ $(n=0,1,2,3,4)$ in **ANS**(CUV=1000) – **ANS**(CUV=0) canceled among 169 diagrams.

	n=0	n=1	n=2	n=3	n=4
sum	3.710749E-26	3.803365E-25	-4.737341E-28	-4.236083E-31	2.888895E-34
max	5.172139E-01	3.560364E-01	4.072901E-01	5.528830E-01	2.814452E-01

- Coefficients of $\left(\tilde{c}_1^\tau\right)^n$ $(n=1,2,3,4)$ in **ANS**(CUV=0) canceled among 521 diagrams.

	n=0	n=1	n=2	n=3	n=4
sum	-6.561361E-01	-1.109906E-28	-1.201544E-31	-4.159490E-34	-1.810261E-36
max	1.106880E-01	1.918886E-03	1.626662E-03	1.425550E-04	9.167856E-05

6. Summary

- We have confirmed the NLG invariance of four decay processes and two scattering processes by introducing the gauge fixing terms of bilinear forms of sleptons, in two kind of renormalization conditions using GRACE/SUSY-loop.