NLO-QCD Event Generator with GRACE

Y.Kurihara(KEK)
GRACE-Group

25/Sep./2010 @CPP2010,
KEK, Tsukuba, Japan

GRACE Author list

J. Fujimoto, T. Ishikawa, M. Jimbo, T. Kaneko, K. Kato, S. Kawabata, T. Kon, Y. Kurihara, M. Kuroda, T. Munehisa, N. Nakazawa, Y. Shimizu, H. Tanaka, T. Ueda, Y. Yasui, Y. Yuasa

Outline

- Introduction
- Matrix Elements
- Loop Library
- ME/PS Matching
- Examples
- Future

Introduction

- What is a "must" for EG at LHC-era?

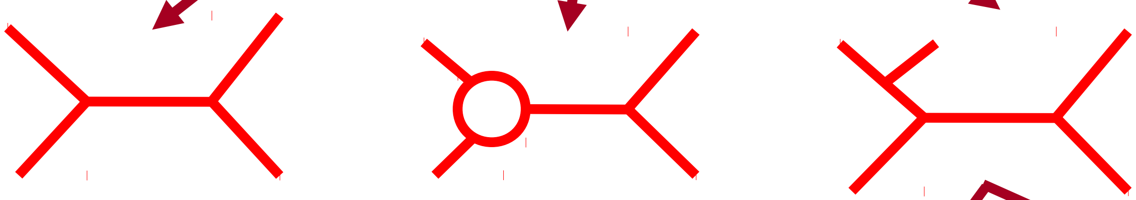
Introduction

- What is a "must" for EG at LHC-era?
  - σ_{tree} x (K-factor) → NLO ME
  - ME/PS Matching
  - Fast & Stable Numerical Calculation
  - High Generation Efficiency
  - Fully Exclusive
  - Non-p Effects if necessary
  - LHA-Compliant Interface

Matrix Elements

Matrix Elements

$$\sigma_{NLO}=\sigma_{tree}+\sigma_{loop}+\sigma_{R}$$



$$=\sigma_{tree}(1 + \delta_V + \delta_{s/c}) + \sigma_{vis}$$

δ_V : Virtual (loop) correction

$\delta_{s/c}$: Soft/Collinear correction

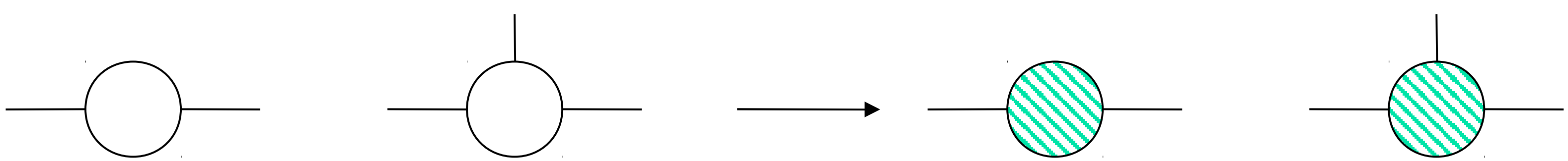
σ_{vis} : Visible jet cross section

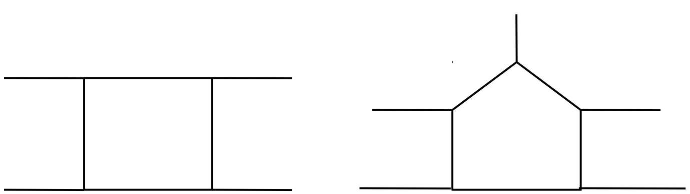
Matrix Elements

- Loop diagrams

$M_V(2\to 2,3)$:Effective vertices

(up to three point)

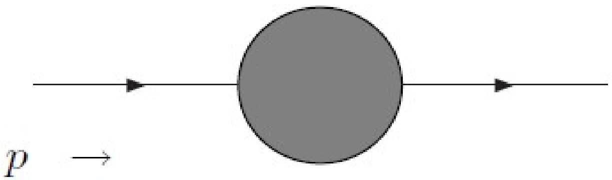




:Numerical calc. by Hel.Amp.

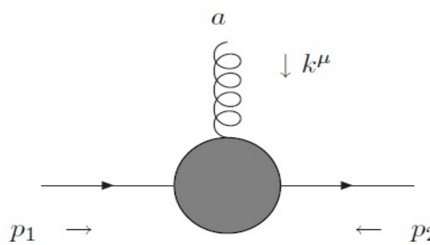
(OP Method by Ishikawa)

Matrix Elements



$$= \Sigma(p^2)$$

on-/off-shell	self-energy
$p^2 \neq 0$	$\Sigma(p^2) = C_F \frac{\alpha_s}{4\pi}(1 - \ln \frac{-p^2}{\mu^2})$
$p^2 = 0$	$\Sigma(0) = C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon_{IR}}$



$$= \Lambda^a_\mu(p_1, p_2, k)$$

$$\begin{aligned} \Lambda_\mu(p_1, p_2, k) &= g\mathbf{T}^a(C_F - \frac{1}{2}C_G)\frac{\alpha_s}{4\pi}(\mathcal{F}_1^I\gamma^\mu + \mathcal{F}_2^I\frac{p_j^\mu \not{k}}{-p_j^2}) \\ &+ g\mathbf{T}^a\frac{1}{2}C_G\frac{\alpha_s}{4\pi}(\mathcal{F}_1^I\gamma^\mu + \mathcal{F}_2^I\frac{p_j^\mu \not{k}}{-p_j^2}) \end{aligned}$$

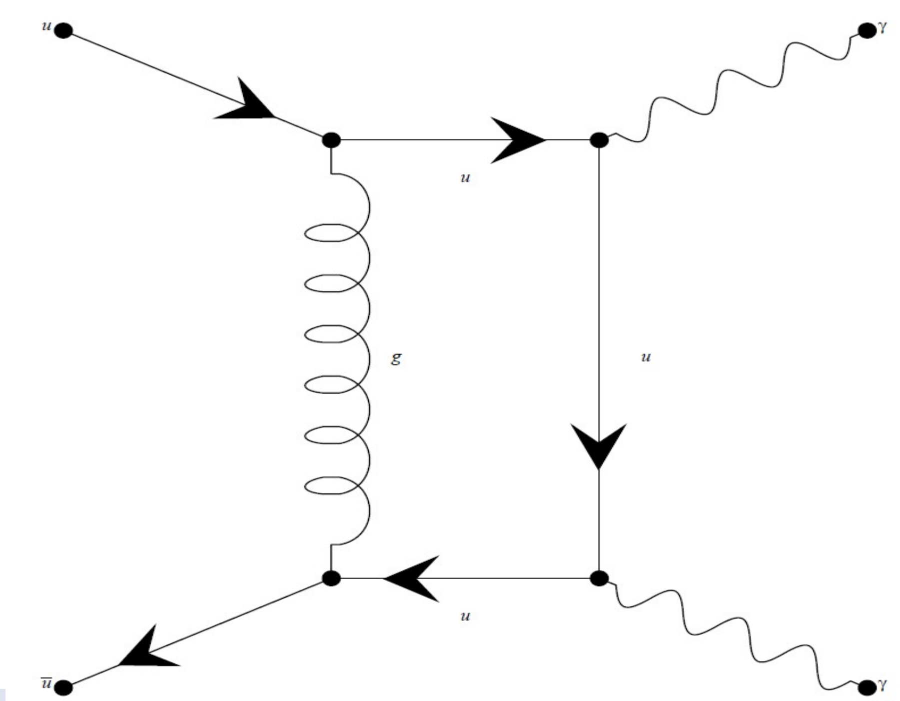
A.5.1 case I

$k^2 = 0$, $p_i^2 = 0$, $p_j^2 = q^2 \neq 0$, and $L = \ln \frac{-q^2}{\mu^2}$, where $(i, j) = (1, 2)$ or $(i, j) = (2, 1)$.

$\mathcal{F}_1^I$	$\frac{2}{\varepsilon_{IR}} + L - 4$
$\mathcal{F}_2^I$	$\frac{4}{\varepsilon_{IR}} + 4L - 10$
$\mathcal{F}_1^{II}$	$-\frac{2}{\varepsilon_{IR}^2} + \frac{3-2L}{\varepsilon_{IR}} + \frac{\pi^2}{6} - L$
$\mathcal{F}_2^{II}$	$-\frac{2}{\varepsilon_{IR}^2} - \frac{2L}{\varepsilon_{IR}} + \frac{12+\pi^2-6L^2}{6}$

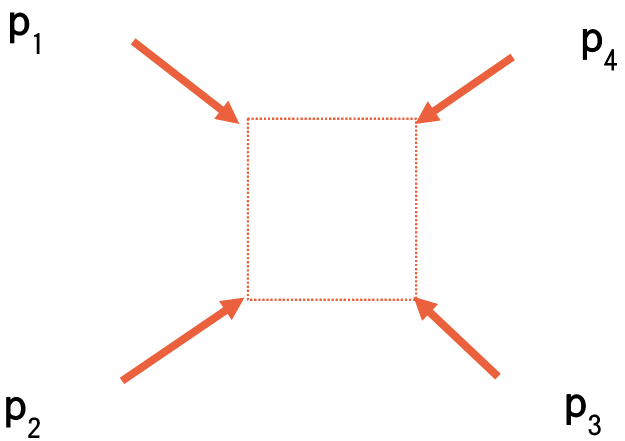
Matrix Elements

```
      do 2002 ih2 =  1, lextrn
      do 2001 ih1 =  1, lextrn
      zsum = 0.0d0
** okikae(8)
      e3e4  = pp4(ep3(1,ih3),ep4(1,ih4))
*----
      zf0=cauq(1)**2*zop3(0,ep4(1,ih4),pf10,ep3(1,ih3),ih2,ih1,1,3,0,
     . 0,ce2,ce1,ps2,ps1)
      zsum =  zsum +  zf0
      agcwrk(ih) = zsum*zbpt
*----- toriaezu no endif
*     endif
*     endif
*     endif
*     endif
      ih = ih + 1
 2001 continue
 2002 continue
 2003 continue
 2004 continue
*---- end of do-loop for helicity
```



Loop Library

Loop Library



$$J_{(4)}(s,t;p_1^2,p_2^2,p_3^2,p_4^2;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x}y^{n_y}z^{n_z}}{D^{2-\varepsilon_{IR}}},$$

$$\begin{aligned} D &= -s\,xz - t\,yw - p_1^2\,xy - p_2^2\,yz - p_3^2\,zw - p_4^2\,xw - i0, \\ w &= 1-x-y-z, \\ s &= (p_1+p_2)^2, \\ t &= (p_1+p_4)^2. \end{aligned}$$

YK Eur.Phys.J.C45(2006)427

Loop Library

$$J_4(s,t;0,0,0,0;n_x,n_y,n_z) = \frac{1}{(4\pi)^2 s\, t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$

$$\times \quad \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(\frac{-t}{s}\right)^{n_x} \frac{B(1+n_z, n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})} \right.$$

$$\times \quad {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$

$$+ \quad \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \left(\frac{-s}{t}\right)^{l} \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} B(1+n_y, l+n_z+\varepsilon_{IR})$$

$$\times \quad \left. {}_2F_1\left(1+l, l+n_z+\varepsilon_{IR}, 1+l+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right],$$

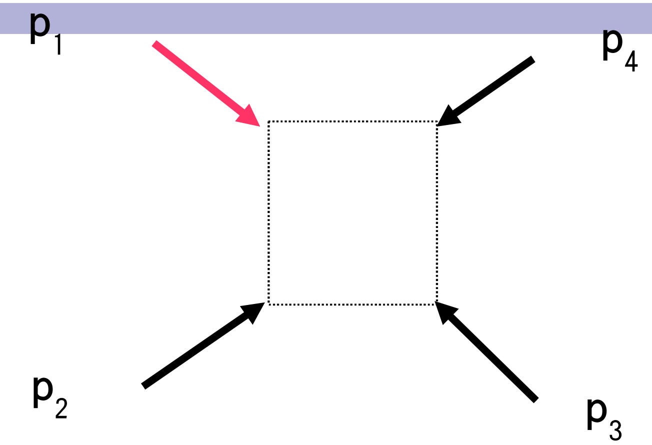
Scalar Integral

$$J_{(4)}(s,t;0,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t} \frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$

$$\times \quad \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1, \varepsilon_{IR}, 1+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1, \varepsilon_{IR}, 1+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)\right]$$

YK Eur.Phys.J.C45(2006)427

Loop Library



$$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z) =$$

$$\frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x}dy\int_0^{1-x-y}dz\frac{x^{n_x}y^{n_y}z^{n_z}}{\left(-xzs-y(1-x-y-z)t-p_1^2xy-i0\right)^{2-\varepsilon_{IR}}}$$

$$\begin{aligned}
= \quad & \frac{1}{(4\pi)^2 s\ t}B(n_x+\varepsilon_{IR},n_y+n_z+\varepsilon_{IR})n_x!\Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR}) \\
\times \quad & \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\left(\frac{-t}{s}\right)^{n_x}\frac{B(1+n_z,n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})}\mathcal{I}^{(1)}\right. \\
+ \quad & \left.\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\sum_{l=0}^{n_x}\frac{(-1)^l B(1+n_y,l+n_z+\varepsilon_{IR})}{\Gamma(l+\varepsilon_{IR})(n_x-l)!}\mathcal{I}_l^{(2)}\right], \qquad (4.2)
\end{aligned}$$

YK Eur.Phys.J.C45(2006)427

Loop Library

$$\mathcal{I}^{(1)} = B(1+n_z, n_x+n_y+\varepsilon_{IR})\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$

$$\begin{aligned}
\mathcal{I}_l^{(2)} &= \sum_{k_1=0}^{n_z} {}_{n_z}C_{k_1} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \sum_{k_2=0}^{n_y+k_1} {}_{n_y+k_1}C_k\ (-1)^{n_y+k_2} \left(\frac{-t}{s}\right) \\
&\times \int_0^1 dw\ \left(1+\frac{\tilde{u}}{\tilde{s}}w\right)^{-(l+1)} \left(1+\frac{\tilde{t}+\tilde{u}}{\tilde{s}}w\right)^{k_2+l-1+\varepsilon_{IR}} \\
&= \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1}\ {}_{n_y+k_1}C_k\ (-1)^{k_1+k_2} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(1+\frac{u}{t}\right)^l \\
&\times \left[\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right. \\
&- \left.\left(\frac{\tilde{p_1}^2}{\tilde{s}}\right)^{l+k_2+\varepsilon_{IR}}\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],
\end{aligned}$$

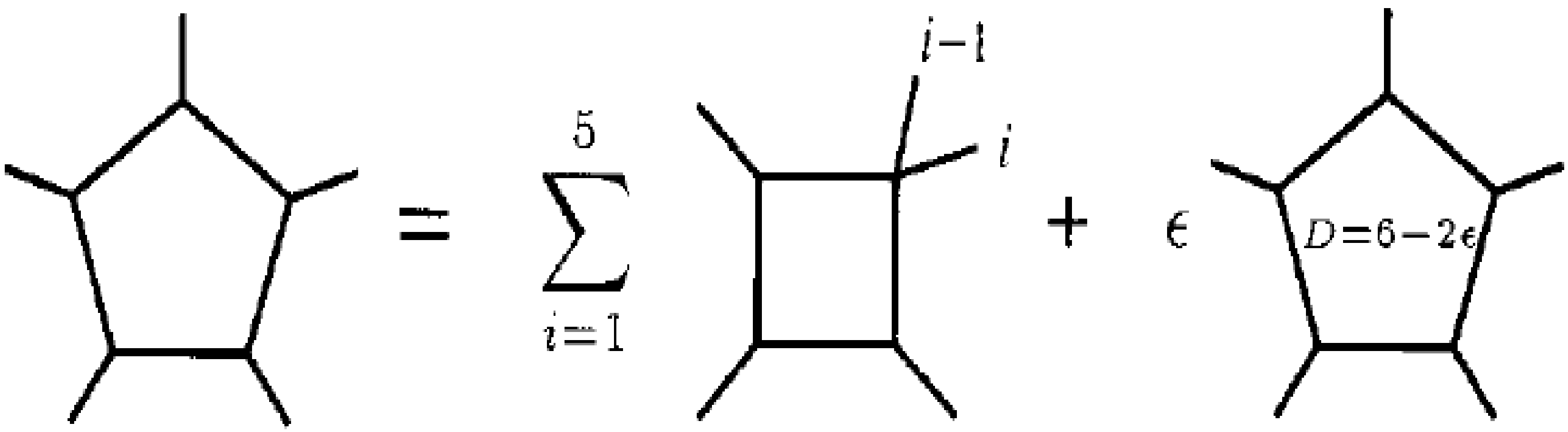
YK Eur.Phys.J.C45(2006)427

Reduction

5->4

Reduction

NLO collections for 3-body final state



Z. Bern, L. Dixon, D. Kosower, Nucl. Physc. **B412**(1994)751.

Reduction method

All massless external legs

	$1/\varepsilon^2$	$1/\varepsilon$	1
GRACE	-0.0550925925925926	0.000952822536342- 0.207985069195991 I	0.279905735189596- 0.00036341419632З I
GOLEM	-0.05509259259	0.0009528225363- 0.20798506912 I	0.2799057352- 0.000363414196З I

Two massive external legs

	$1/\varepsilon^2$	$1/\varepsilon$	1
GRACE	-0.0000433333333333333	0.000047107451540878- 0.000139272250775022 I	0.000229724271506527+ 0.000173645728826450 I
GOLEM	-0.00004333333333	0.00004710745154- 0.0001392722508 I	0.0002297242715+ 0.0001736457288 I

Reduction method

Three massive external legs

	$1/\varepsilon^2$	$1/\varepsilon$	1
GRACE	0	0.0000763703127368405+ 7.0781353201340*10^{-6} I	−0.0000355207656816 58+ 0.000284927385986234 I
GOLEM	0	0.00007637031273+ 7.078135320*10^{-6} I	0.2799057352- 0.0003634141963 I

Four massive external legs

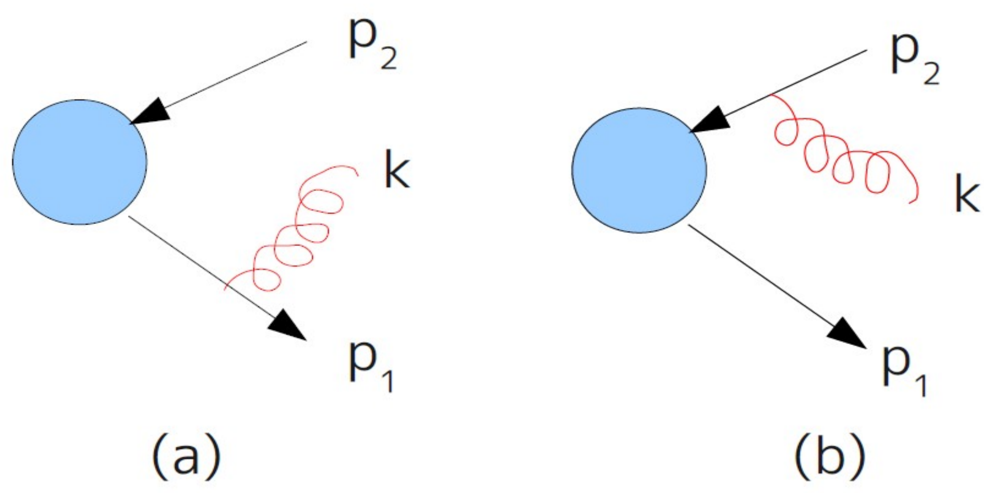
	$1/\varepsilon^2$	$1/\varepsilon$	1
GRACE	0	0.000196907008457362	0.000404112408707748+ 0.000639438626350558 I
GOLEM	0	0.0001969070085	0.0004041124087+ 0.0006394386258 I

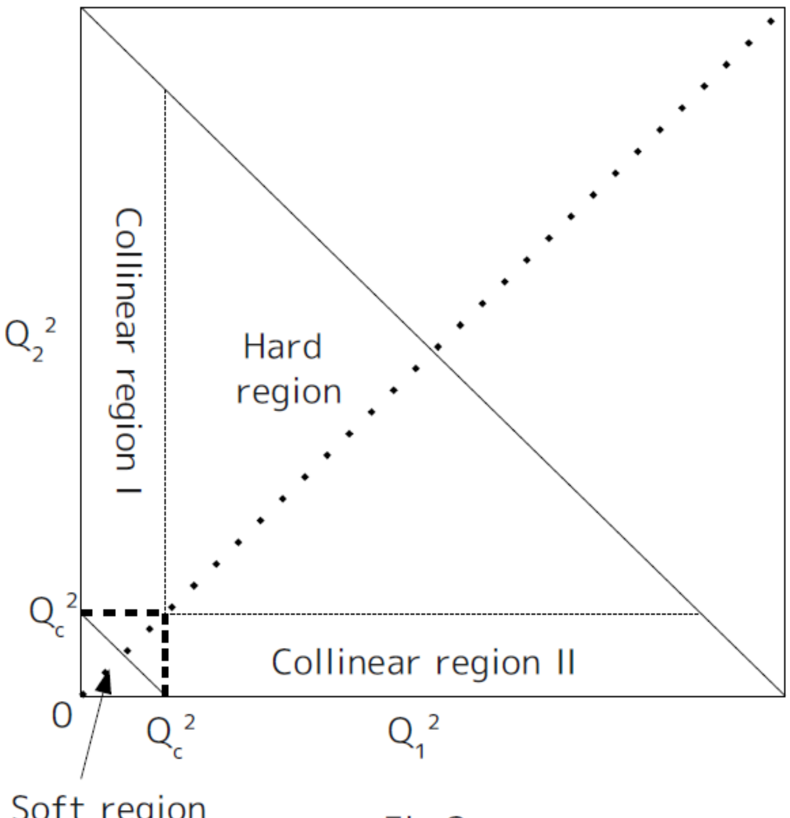
ME/PS Matching

Soft/Collinear correction (final color)

Two types of div.

- Collinear div.
- Soft div.





Lorentz inv./Process indep.

- Phase space slicing
- Subtraction
- Slicing/LLL-Subtraction hybrid

Soft/Collinear correction (final color)

Collinear Part

$$\sigma_{n+1} = \frac{1}{(flux)} \int d\Phi_{n+1} |\mathcal{M}|^2$$

$$\approx \frac{1}{(flux)} \int d\Phi_{n+1} |\mathcal{M}_0|^2 \frac{R_I}{(q_1^2)^2}$$

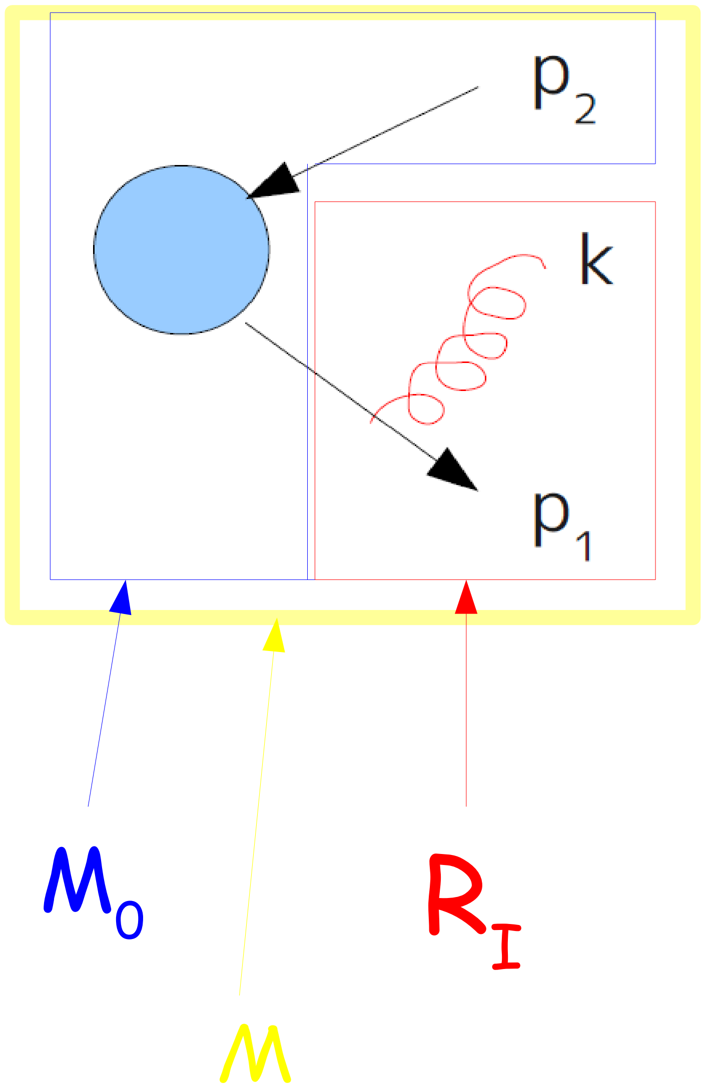
$$= \frac{1}{(flux)} \int d\Phi_n |\mathcal{M}_0|^2 \int \frac{dq_1^2}{2\pi (q_1^2)^2} \int d\Phi_2 R_I$$

$$= \frac{1}{(flux)} \int d\Phi_n |\mathcal{M}_0|^2 \otimes \delta_{col},$$

$$\delta_{col} \equiv \int \frac{dq_1^2}{2\pi (q_1^2)^2} \int d\Phi_2 R_I$$

$$R_I = 2 g_s^2 \frac{k_T}{x(1-x)} (P(x) + (1-x)\epsilon_{IR})$$

$$P(x) = \frac{1+x^2}{1-x}$$



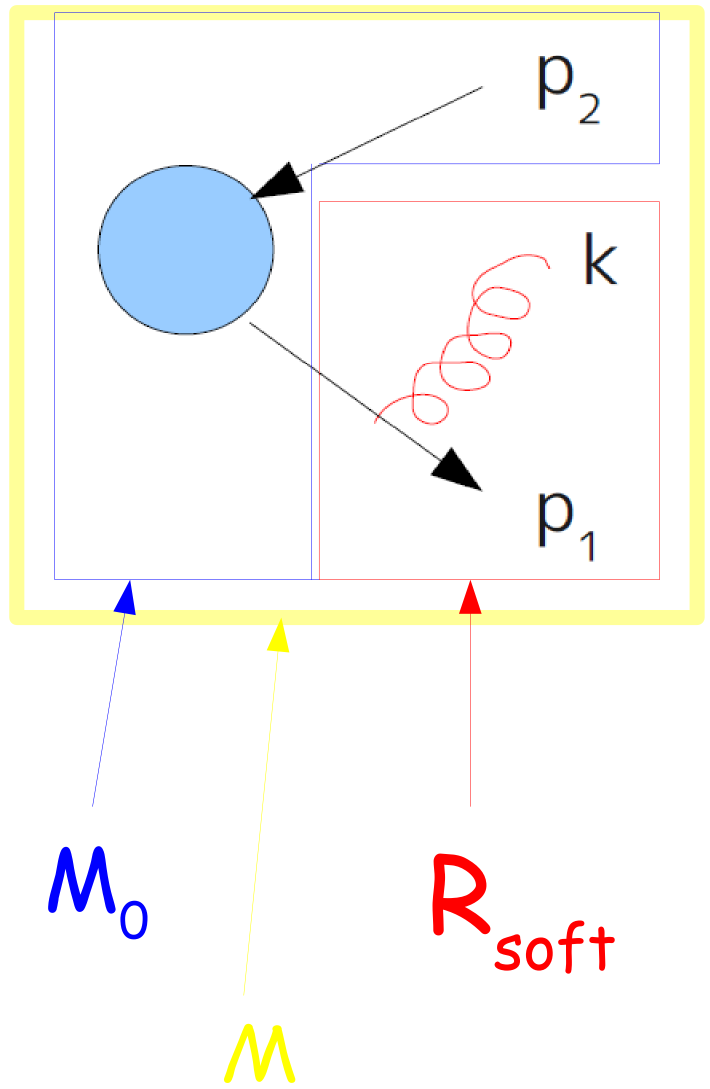
Soft/Collinear correction (final color)

Soft Part

$$\sigma_{n+1} = \frac{1}{(flux)} \int d\Phi_{n+1} |\mathcal{M}|^2$$

$$\approx \frac{1}{(flux)} \int d\Phi_n |\mathcal{M}_0|^2 \otimes \delta_{soft},$$

$$\delta_{soft} \equiv \int \frac{dk_0}{\pi} \int d\Phi_2 \frac{s R_{soft}}{q_1^2 q_2^2}$$



Soft/Collinear correction (final color)

Collinear Part

$$\delta_{col} = \frac{\alpha_s}{\pi}\frac{1}{\epsilon_{IR}^2} + \frac{\alpha_S}{4\pi}\left(-3+4\log\left(Q_c^2/\mu_F^2\right)\right)\frac{1}{\epsilon_{IR}}$$
$$-\frac{\alpha_s}{4\pi}\left(-7+\pi^2+3\log\left(Q_c^2/\mu_F^2\right)-\log^2\left(Q_c^2/\mu_F^2\right)\right)$$

Soft Part

$$\delta_{soft} = -\frac{\alpha_s}{\pi}\frac{1}{\epsilon_{IR}^2} - \frac{\alpha_S}{\pi}\left(\log\left(Q_c^2/s\right)+\log\left(Q_c^2/\mu_F^2\right)\right)\frac{1}{\epsilon_{IR}}$$
$$+\frac{\alpha_s}{12\pi}\left(\pi^2-6\left(\log\left(Q_c^2/s\right)+\log\left(Q_c^2/\mu^2\right)\right)^2\right)$$

Soft/Collinear correction (final color)

Examples $Z \to d\bar{d}$

Virtual correction

$$\delta_v = -\frac{\alpha_s}{\pi}\frac{1}{\epsilon_{IR}^2} + \frac{\alpha_S}{2\pi}\left(3 - 2\log(s/\mu_F^2)\right)\frac{1}{\epsilon_{IR}}$$
$$+\frac{\alpha_s}{2\pi}\left(-8 + \frac{7}{6}\pi^2 + 3\log(s/\mu_F^2) - \log^2(s/\mu_F^2)\right)$$

$$\Gamma_{Z\to d\bar{d}} = \Gamma_0(1+\delta_{\alpha s}),$$
$$\delta_{\alpha s} = \delta_v + 2\delta_{col} + \delta_{soft}$$

Result $$\delta_{\alpha s} = \frac{\alpha_s}{2\pi}\left(-1 + \pi^2/3 - 3\log(Q_c^2/s) - 2\log^2(Q_c^2/s)\right)$$

Parameters

m_z	91.187 GeV	m_d	0
α_s	0.12	α	1/128

Soft/Collinear correction (final color)

$Z \to d\bar{d}$

Numerical esult

Tree : $\Gamma_0 = 0.378458\ GeV$

Known result : $\Gamma_{\alpha s} = \Gamma_0 \dfrac{\alpha_s}{\pi} = 0.01445\quad GeV$

$Q_c^2\ GeV$	Hard	v+s+c	total
10^0	1.3017	-1.2872	0.0145
10^{-1}	2.1384	-2.1240	0.0144
10^{-2}	3.1794	-3.1651	0.0143
10^{-3}	4.4249	-4.4107	0.0142
10^{-4}	5.8747	-5.8606	0.0141
10^{-5}	7.5289	-7.5150	0.0140

Soft/Collinear correction (final color)

$$e^+ e^- \to \gamma \to u\bar{u}$$

$\sqrt{s}=100GeV$

Q_c^2 GeV	Hard	v+s+c	total
10^0	47.689	-47.181	0.509
10^{-1}	77.579	-77.070	0.508
10^{-2}	114.63	-114.12	0.509
10^{-3}	158.84	-158.33	0.511
10^{-4}	210.21	-209.70	0.511
10^{-5}	268.75	-268.21	0.517
$\sigma_0=13.2588$ pb		$\sigma_0\ \alpha_s/\pi=0.50645$ pb	

$\sqrt{s}=1000GeV$

Q_c^2 GeV	Hard	v+s+c	total
10^0	1.1463	-1.1412	0.00509
10^{-1}	1.5884	-1.5833	0.00511
10^{-2}	2.1021	-2.0970	0.00511
10^{-3}	2.6875	-2.6823	0.00517
10^{-4}	3.3444	-3.3392	0.00518
10^{-5}	4.0730	-4.0678	0.00522
$\sigma_0=0.132588$ pb		$\sigma_0\ \alpha_s/\pi=0.0050645$ pb	

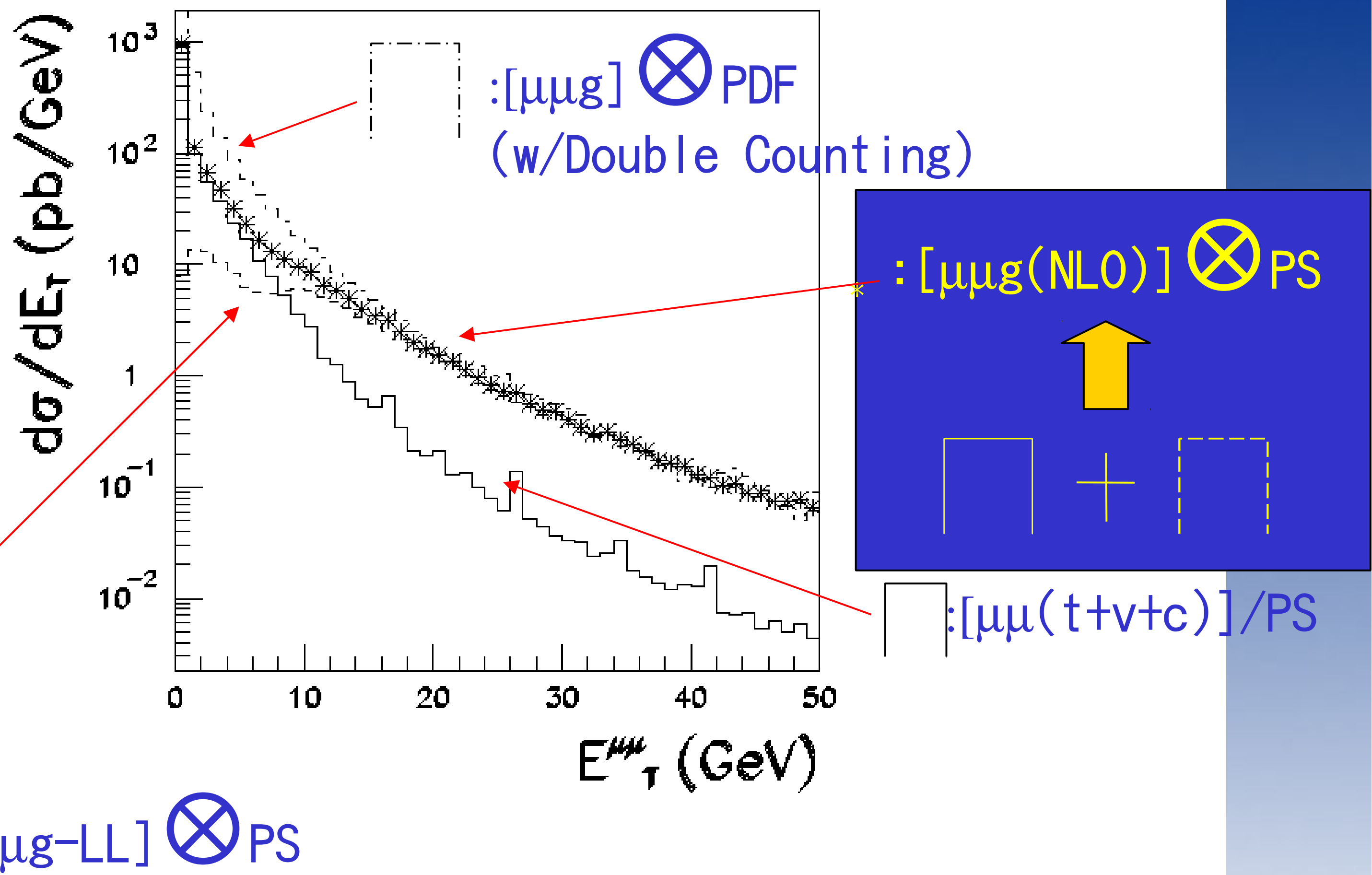
Examples

GRACE for LHC : NLO Generator

ME⇔PS Matching@NLO

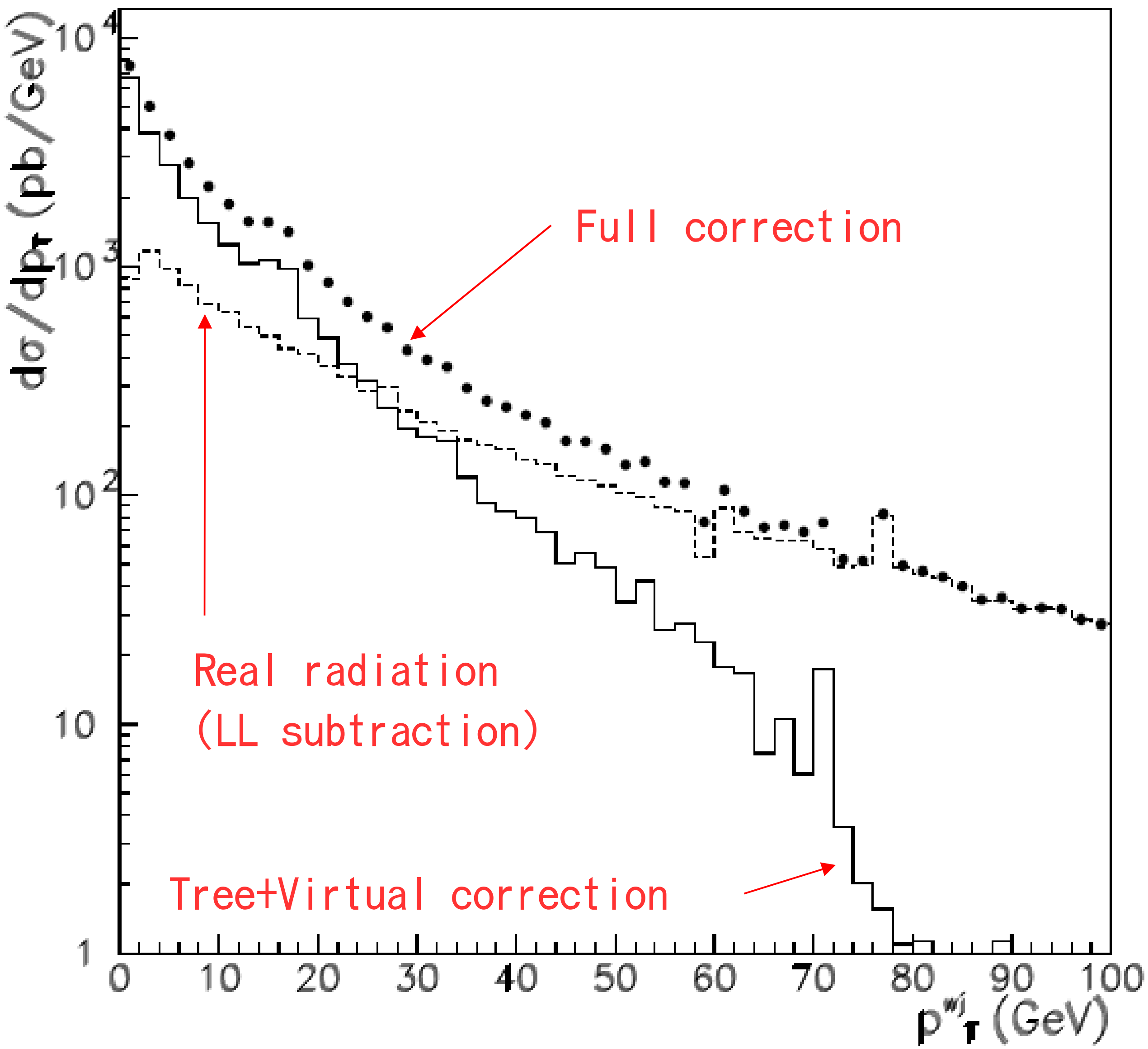
- Process :
  uu→$\mu^+\mu^-$(+gluon)
  in pp collision

Cuts:
- $\sqrt{s_{\mu\mu}}$>40GeV
- k_T^g> 1 GeV



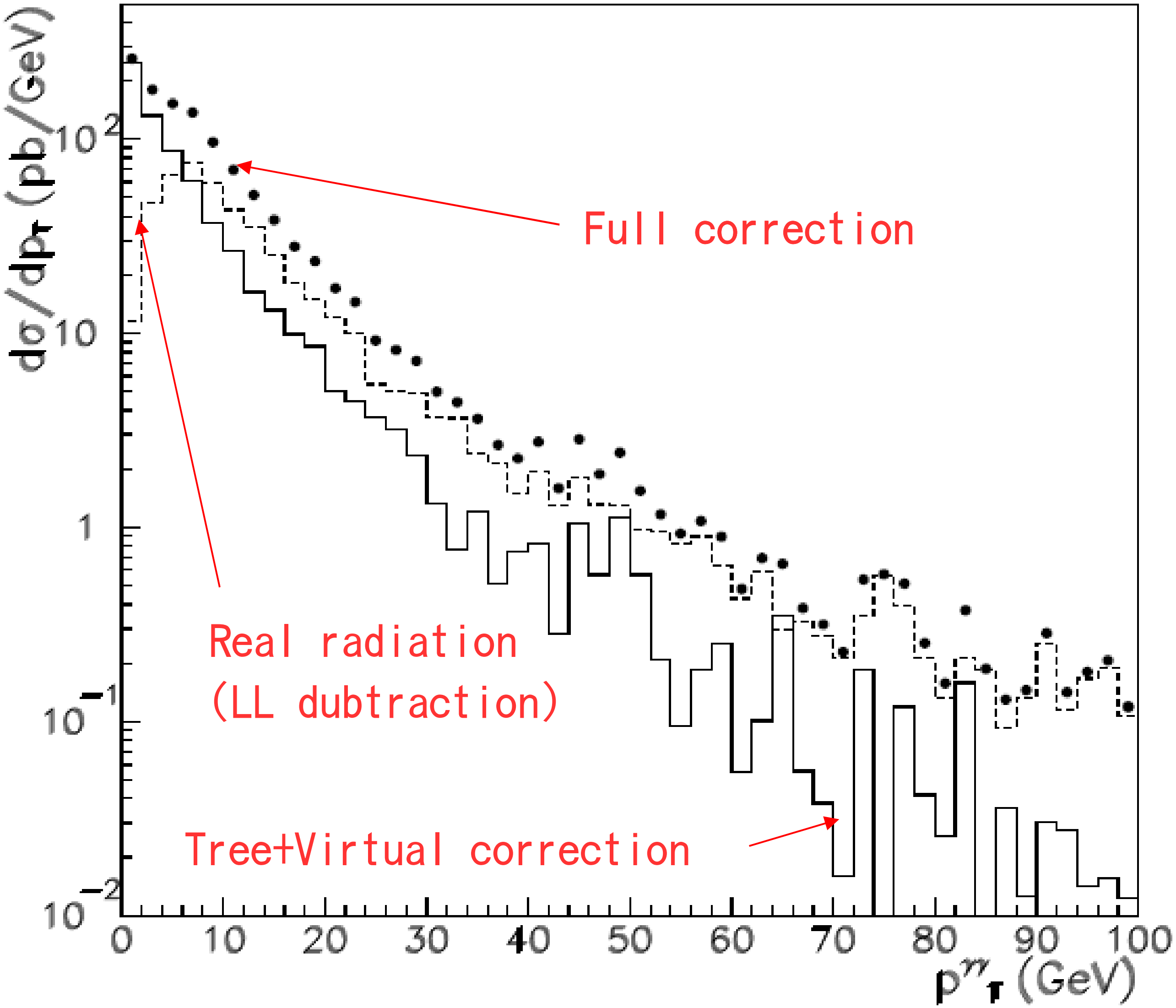
GRACE for LHC : NLO Generator

Transverse momentum distribution of W-jet



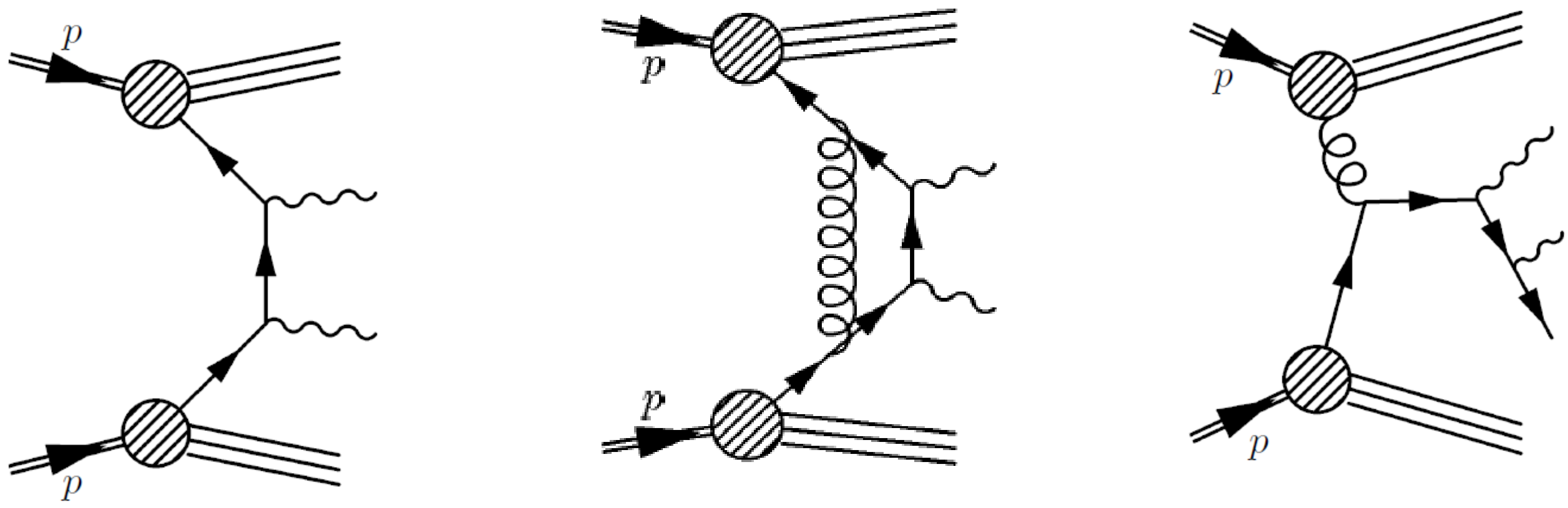
GRACE for LHC : NLO Generator

Transverse momentum distribution of $\gamma\gamma$



GRACE for LHC : $\gamma\gamma$ Generator

Direct Processes

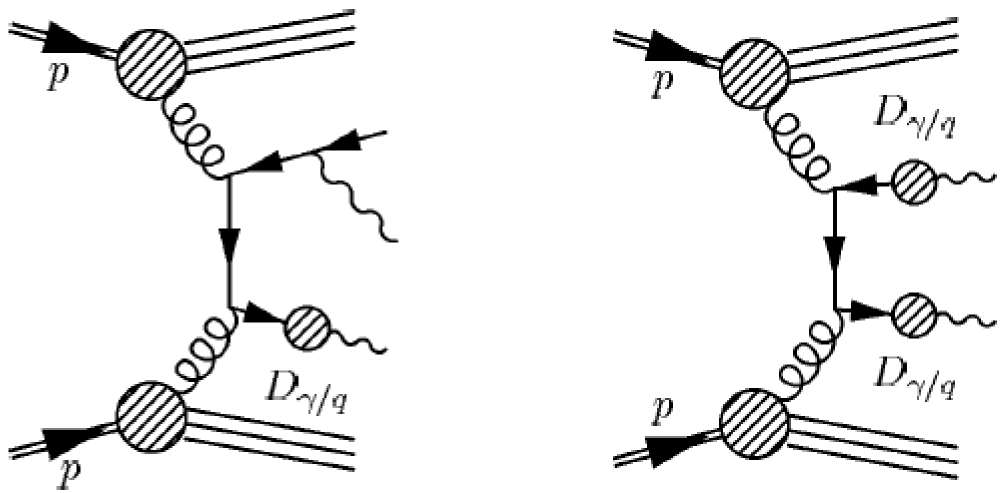


Lowest Process | Loop Correction | Real Radiation

- Diphox
  - T.Binoth,J.Ph.Guillet,E.Pilon,M.Werlen
  - Eur.Phys.J.C16,311(2000)

GRACE for LHC : NLO Generator

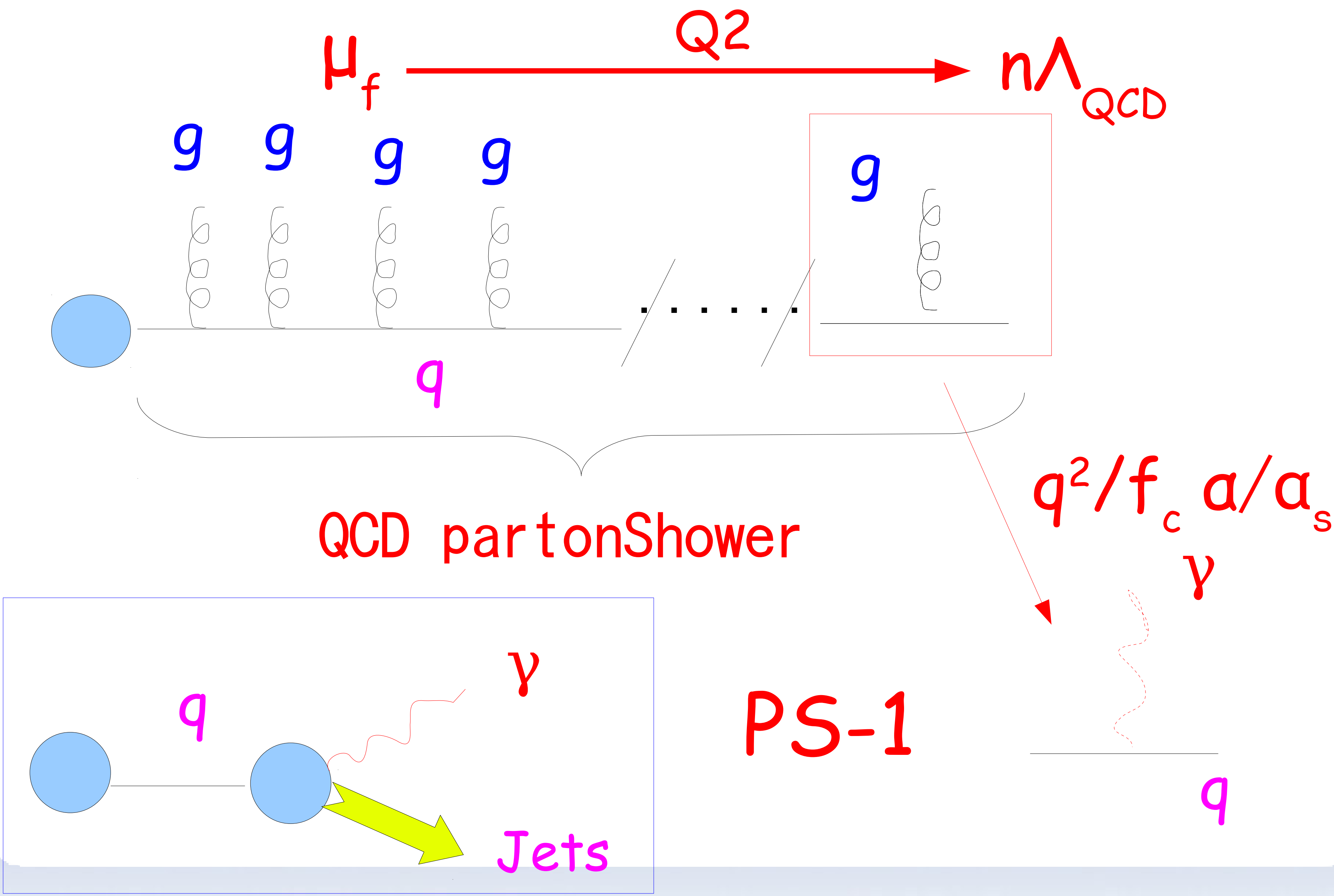
Fragmentation Processes



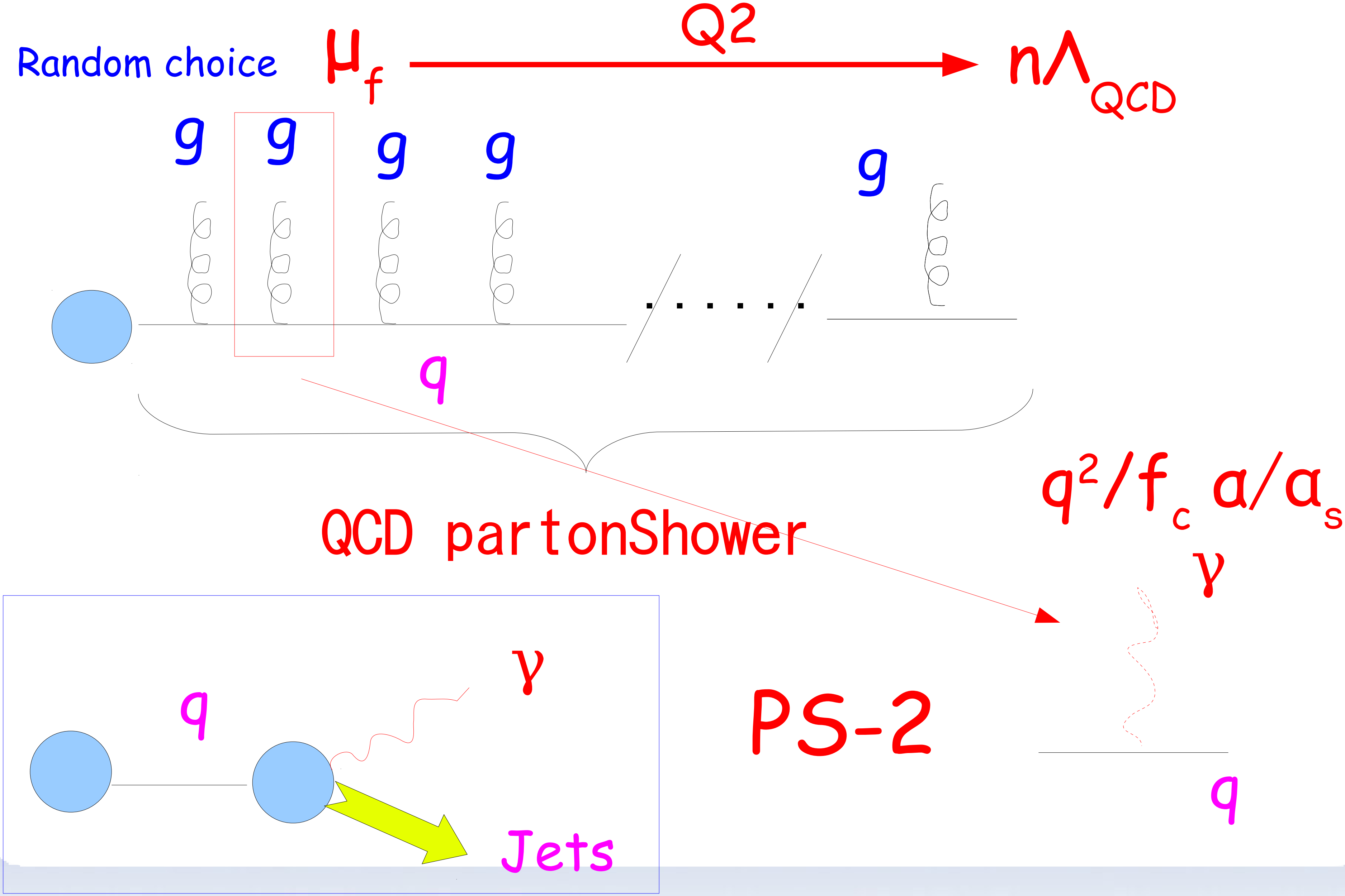
single fragmentation   double fragmentation

- DIPHOX
  - Fragmentation Function
  - Inclusive Jet → No Event-Generation
- GR@PPA
  - Parton Shower (QCD/QED Mixed)
  - Fully Exclusive→Event-Generation

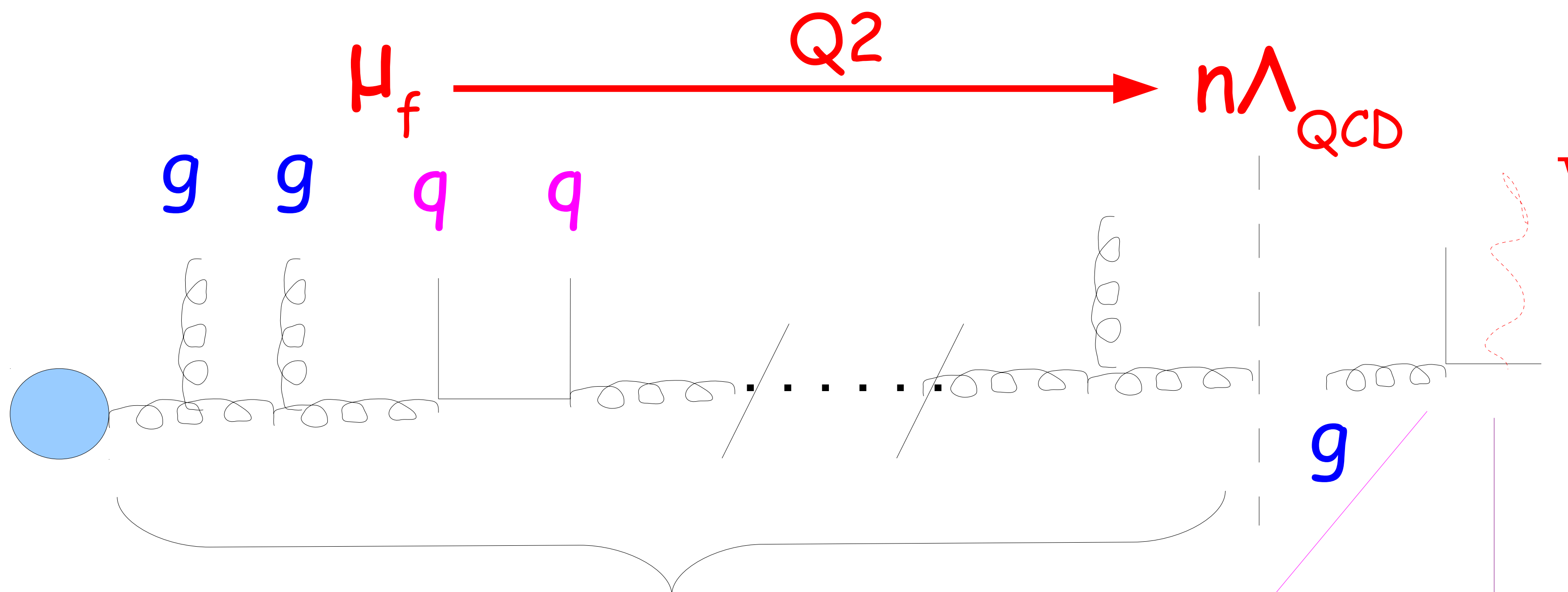
GRACE for LHC : QCED PS



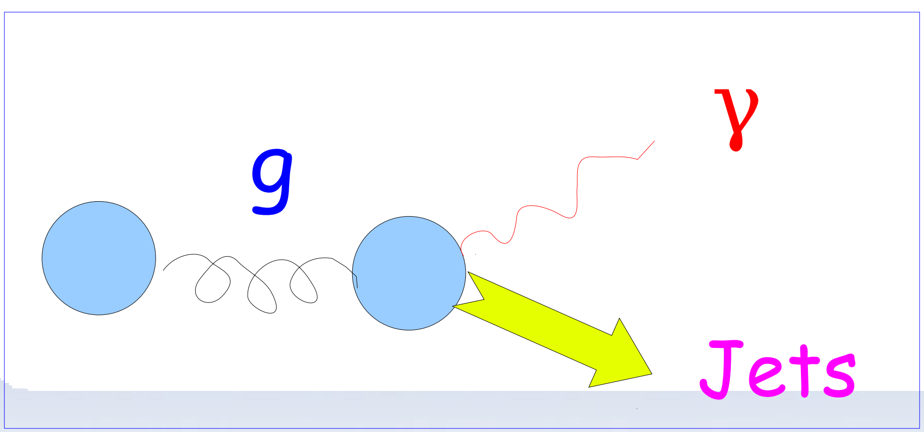
GRACE for LHC : QCED PS



GRACE for LHC : QCED PS



QCD partonShower



4%

g→qq

$$\frac{\alpha}{2\pi}\frac{1}{k_T^2}\frac{1+x^2}{1-x}$$

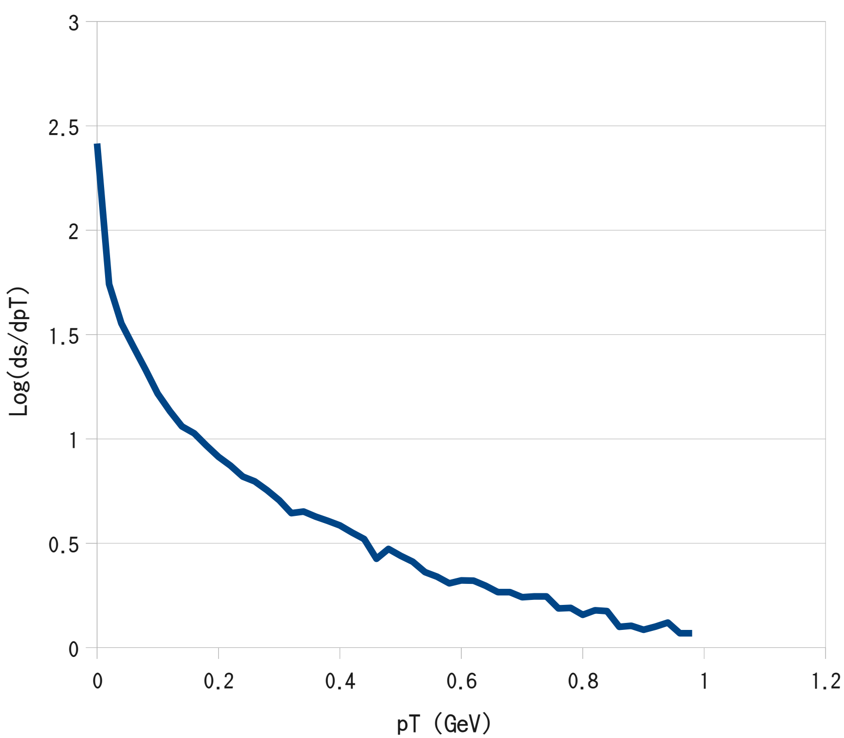
GRACE for LHC : QCED PS

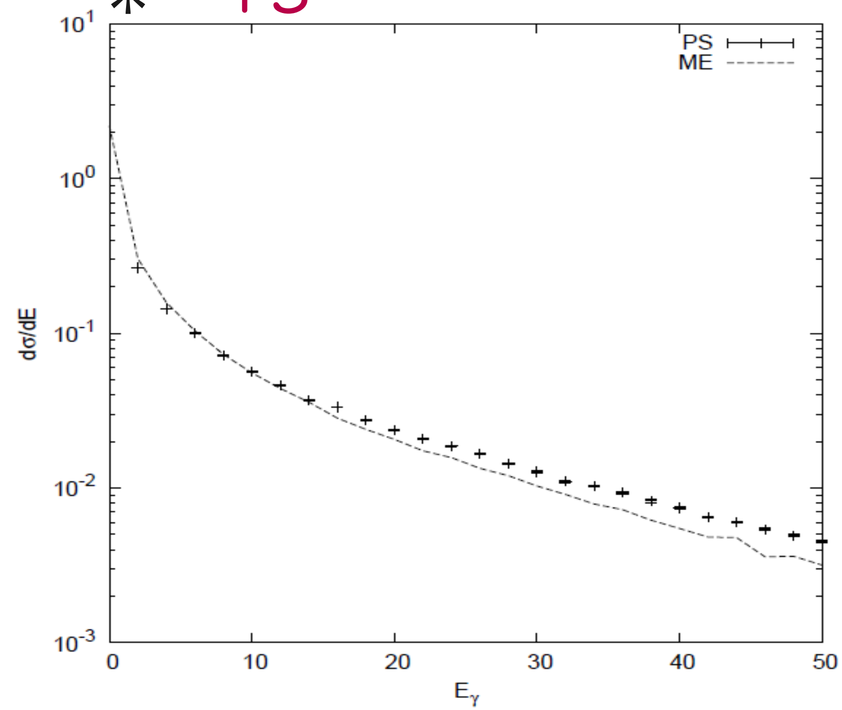
$Q2=(50\ \mathrm{GeV})\sim(5\Lambda_{QCD})$, E_{gluon} = 100GeV

—— PS

—— ME calc. uu →g* → dd γ

* PS





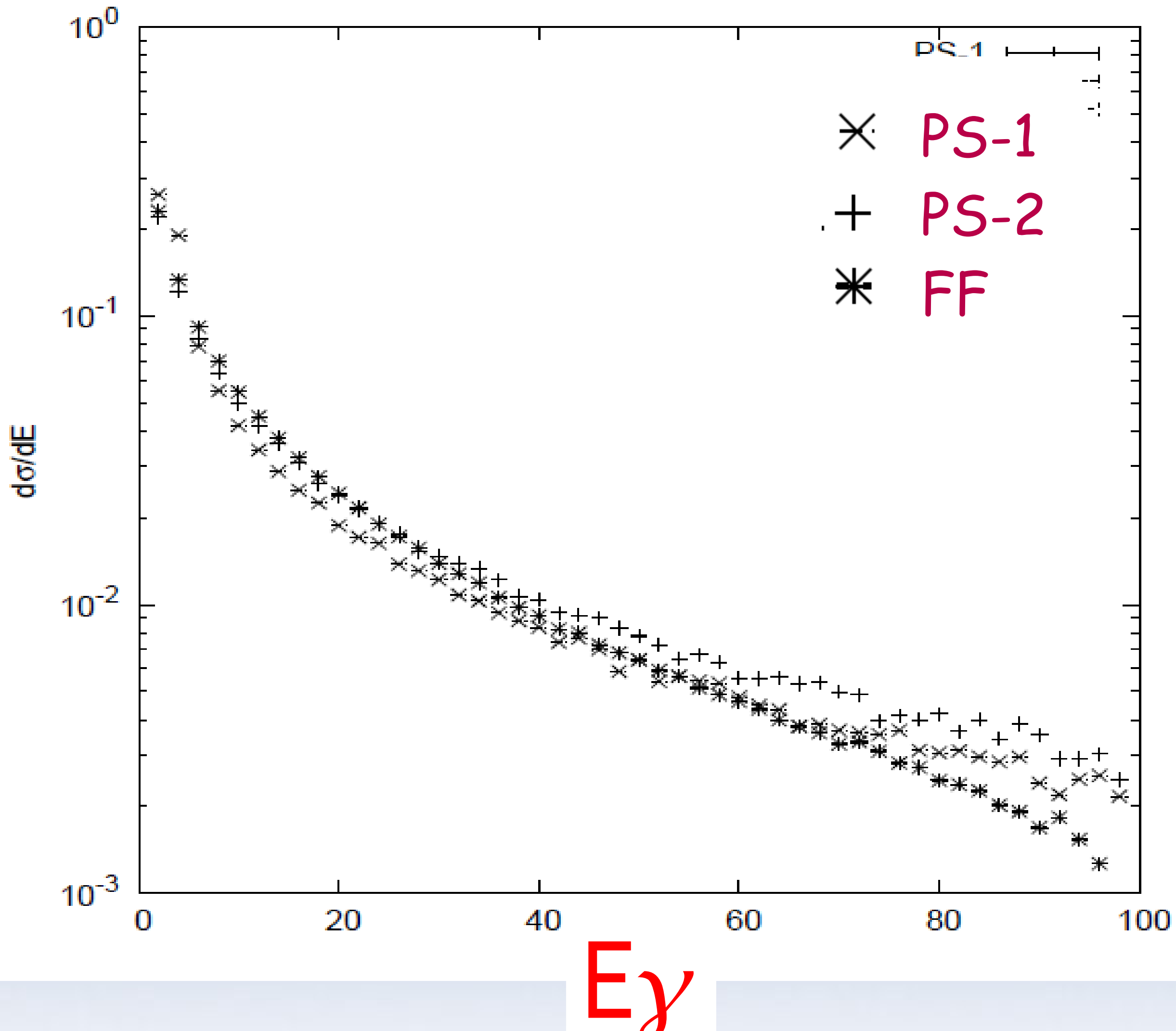
jet pT

γ energy

GRACE for LHC : QCED PS

Comparison w/ Fragmentation Function Method

γ energy



Future

- GR@PPA 2.8 → 3.0 : NLO + QECD PS Full Exclusive unweighted Event Generator
- Single-Top Process,...