Application of reduction and Heavy quark mass effects on photon structure functions

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Outline

- Introduction and Motivation

- How to calculate the structure functions in parton model
    - LO diagrams
    - NLO diagrams(One-gluon corrections)

    Virtual gluon corrections

    Real gluon emissions

- Summary

I. Introduction

- Future linear collider experiment (ILC)

Two-photon process $e^+e^- \to (e^+e^-\gamma\gamma) \to e^+e^-X$

Viewed as a deep-inelastic electron-photon scattering when $P^2 \ll Q^2$

We can study the structures of photon

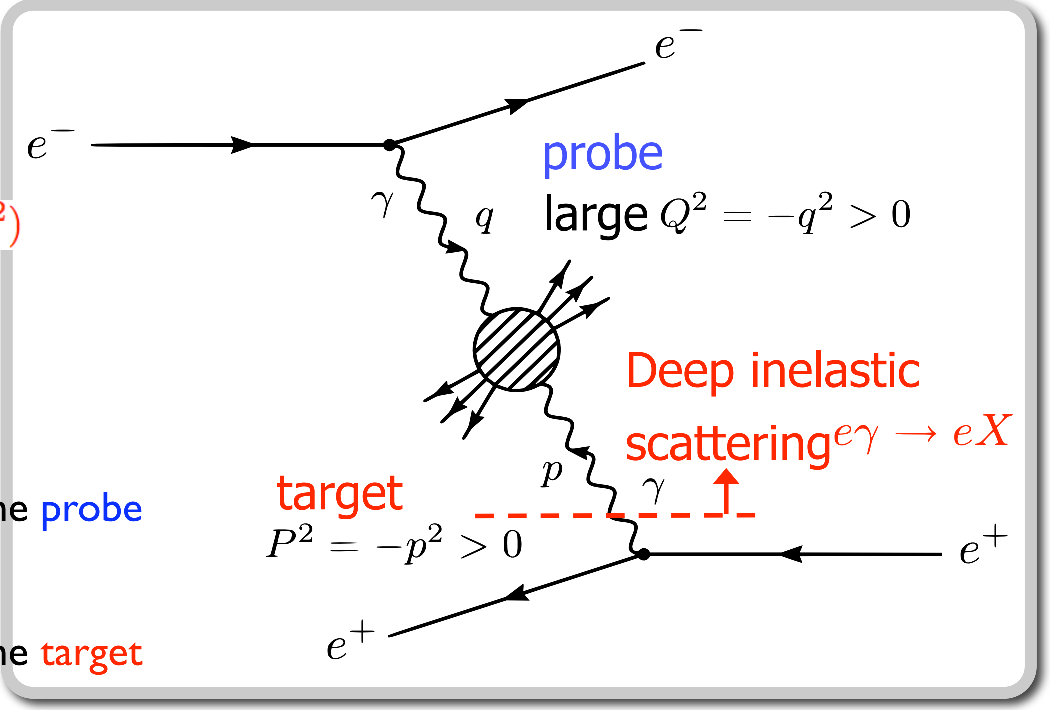
$F_2^\gamma(x, Q^2, P^2)$ and $F_L^\gamma(x, Q^2, P^2)$

$x = \dfrac{Q^2}{2p \cdot q}$:Bjorken variable

$0 \leq x \leq 1$

$-Q^2 = q^2 \leq 0$:mass squared of the probe photon

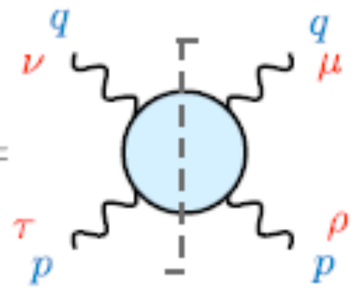
$-P^2 = p^2 \leq 0$:mass squared of the target photon



Photon Structure Functions

Absorptive part of photon-photon forward scattering

$$(4\pi\alpha)W_{\mu\nu\rho\tau} = \frac{1}{2\pi}\sum_{k=2}^{\infty} dPS^k M^*_{\mu\rho} M_{\nu\tau}$$

$W_{\mu\nu\rho\tau} =$ 

- Structure functions

For the real photon target

Budnev-Ginzburg-Meledin-Serbo('75)

$$W^{\mu\nu\rho\tau} = (T_{TT})^{\mu\nu\rho\tau} W_{TT} + (T^a_{TT})^{\mu\nu\rho\tau} W^a_{TT} + (T^\tau_{TT})^{\mu\nu\rho\tau} W^\tau_{TT} + (T_{LT})^{\mu\nu\rho\tau} W_{LT}$$

$$F_2^\gamma = x(W_{TT} + W_{LT}), \quad F_L^\gamma = xW_{LT}$$

$$g_1^\gamma = W^a_{TT}$$

$$W_3^\gamma = W^\tau_{TT}$$

- Projection operator

$$(T_{TT})^{\mu\nu\rho\tau} = R^{\mu\nu} R^{\rho\tau}$$

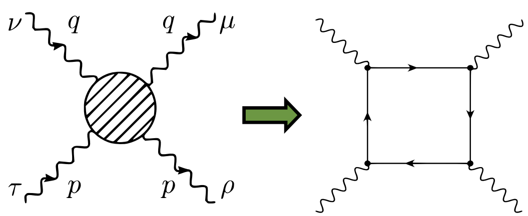
$$(T_{LT})^{\mu\nu\tau\rho} = k_1^\mu k_1^\nu R^{\rho\tau}$$

$$R^{\mu\nu} = -g^{\mu\nu} + \frac{1}{(p\cdot q)^2}[p\cdot q(q^\mu p^\nu + q^\nu p^\mu) - q^2 p^\mu p^\nu]$$

$$k_1^\mu = \frac{\sqrt{-q^2}}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)$$

F_2^γ in Perturbative QCD

Simple parton model



- For real photon target $(P^2 \approx 0)$ $P^2 \ll Q^2$

$$F_2^\gamma(x,Q^2) = \alpha \left[\boxed{\frac{1}{\alpha_s(Q^2)}} A + B + B' + \mathcal{O}(\alpha_s) \right]$$

(LO) (NLO) Hadronic piece

$\sim \ln \frac{Q^2}{\Lambda^2}$

Point-like contribution dominates $\sim \ln Q^2$

OPE+RGE lowest order in α

Witten (1977)

Bardeen-Buras (1979)

Walsh-Zerwas (1973)

NNLO extension Moch-Vermaseren-Vogt (2002, 2006)

- For highly virtual photon target $(\Lambda^2 \ll P^2 \ll Q^2)$

$$F_2^\gamma(x,Q^2,P^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} \tilde{A} + \tilde{B} + \mathcal{O}(\alpha_s) \right]$$

Λ : QCD scale parameter

(LO) (NLO) Uematsu-Walsh (1981,1982)

Hadronic piece can also be dealt with perturbatively

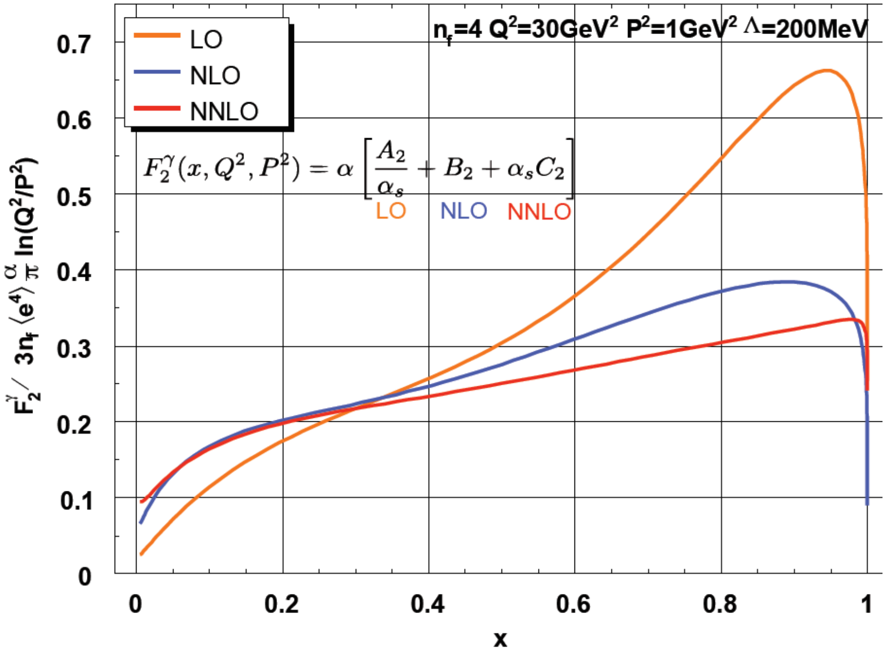
Definite prediction of F_2^γ , its shape and magnitude, is possible

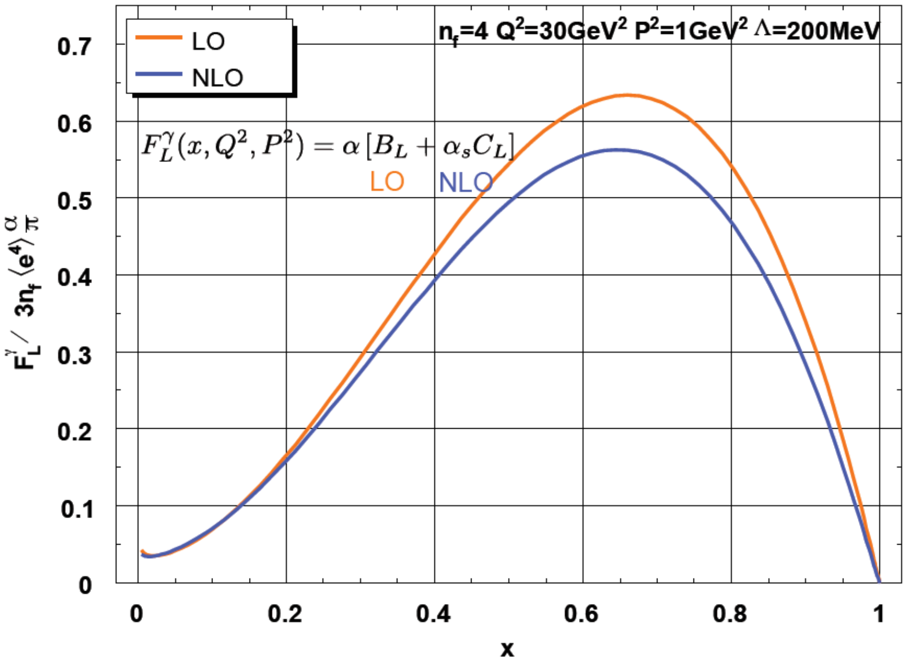
- NNLO $(\alpha\alpha_s)$ extension Ueda-Sasaki-Uematsu (PRD75,114009(2007))

Motivated by the calculation of 3-loop anomalous dimensions
Vogt-Moch-Vermaseren (2004,2006)

Photon Structure functions

$\Lambda^2 \ll P^2 \ll Q^2$





Ueda-Sasaki-Uematsu (2007)

- NNLO QCD analysis performed with 3-loop splitting fns. and 2-loop coefficient fns. for massless quarks

The heavy quark effects in DIS processes

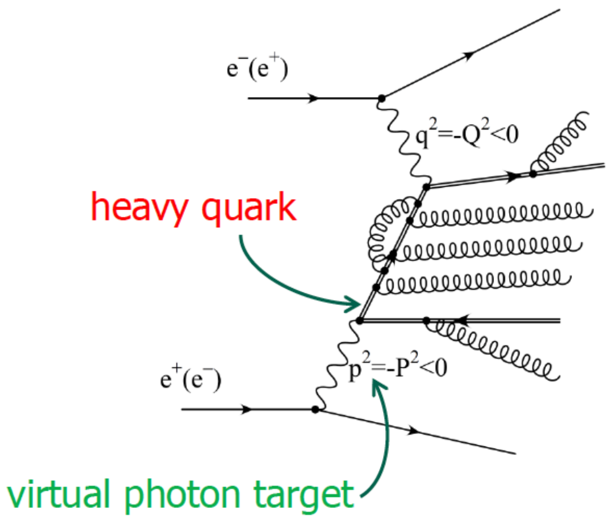
- In the case of nucleon target, the heavy quarks are treated as being absent in the intrinsic parton components but radiatively generated from the gluon and light quarks

- For photon target, the heavy quarks are treated in the same way as light quarks and generated from the photon target (and also from gluon and light quarks)

- Heavy quark mass effects for the real photon case are studied up to LO

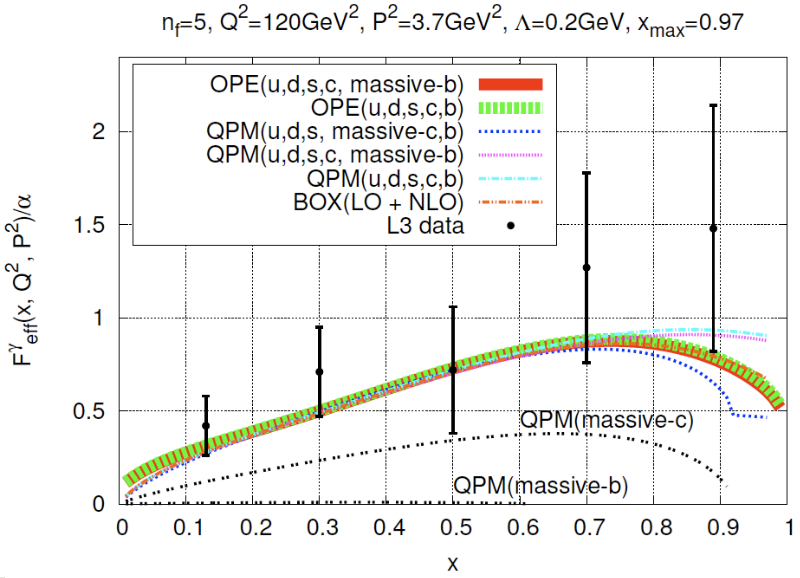
  M. Gluck, E. Reya & A. Vogt,

  F. Cornet, P. Jankowski & M. Krawczyk, A. Lorca, …..



The heavy quark effects in DIS processes

- Heavy quark mass effects for the virtual photon target are studied up to NLO in the framework of OPE and mass independent renormalization group method



Kitadono-Sasaki-Ueda-Uematsu

PTP121,495 (2009), PRD81,074029 (2010)

NLO QCD prediction vs. L3 data

- The kinematical threshold effects

$$x^H_{\max} = \frac{1}{1 + \frac{4m_H^2}{Q^2} + \frac{P2}{Q^2}}$$

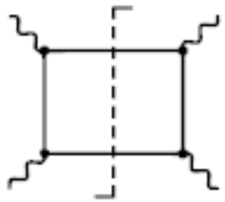
In order to see the kinematical threshold effects,

$$(p+q)^2 \geq 4m_H^2$$

we analyze the heavy quark effects in the parton model

The heavy quark effects in the parton model

- The heavy quark mass effects on photon structure fns. in parton model were calculated up to LO

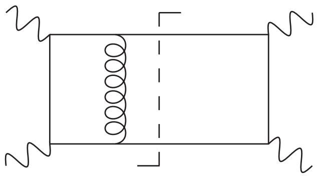
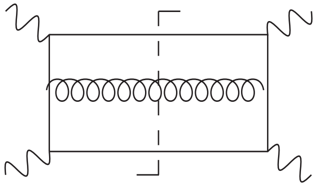


Budnev-Ginxburg-Meledin('75)

Gluck-Reya-Schienbein('01)

Sasaki-Soffer-Uematsu ('02)

- We investigate the heavy quark mass effects for on-shell photon with one-gluon corrections in the parton model

 ....  ....

The heavy quark effects in the parton model

Also we check (with gluon corrections)

- Positivity constraint still holds ?

$$|W^{\tau}_{TT}| \leq (W_{TT} + W^{a}_{TT})$$

Sasaki-Soffer-Uematsu ('02)

- The sum rule still holds ?

$$\int_0^{x_{max}} dx\ W^{a}_{TT}(x, Q^2) = 0$$

- The behaviour of $W^{\tau}_{TT}(x, Q^2)$

Twist-2-quark operators do not contribute to W^{τ}_{TT} Sasaki ('80)

How to calculate

- Using Cutkosky rule $(2\pi)\delta^{(+)}(k^2-m^2) = \frac{i}{k^2-m^2+i\epsilon} - \frac{i}{k^2-m^2-i\epsilon}$

absorptive parts of photon-photon scattering are transformed into box diagrams

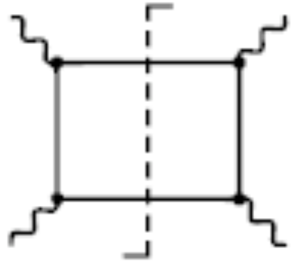
- Take trace of box diagrams and apply the projection operators
- Structure functions are written in a linear combination of many scalar integrals
- By reduction method scalar integrals are expressed as a linear Combination of fewer master integrals

IBP identity $\int d^D l \frac{\partial}{\partial l^\mu}\Big(\frac{\eta^\mu}{D_1^{\nu_1}D_2^{\nu_2}D_3^{\nu_3}D_4^{\nu_4}}\Big) = 0$

- Discontinuities of the master integrals are evaluated

- In practice, $W_{TT}(x,Q^2)$ is calculated in LO

Leading order calculation



$$\int \frac{d^D k_1}{(2\pi)^{D-1}} \delta^{(+)}(k_1^2 - m^2) \int \frac{d^D k_2}{(2\pi)^{D-1}} \delta^{(+)}(k_2^2 - m^2)(2\pi)^D \delta^{(D)}(k_1 + k_2 - p - q)$$

- Cutkosky rule $(2\pi)\delta^{(+)}(k^2 - m^2) = \dfrac{i}{k^2 - m^2 + i\epsilon} - \dfrac{i}{k^2 - m^2 - i\epsilon}$

Box diagram

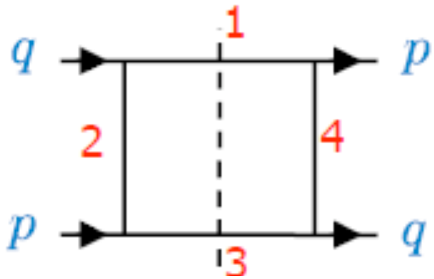
$$A_{\mu\nu\rho\tau} = \int \frac{d^D k_1}{(2\pi)^D} e^4 (-) \mathrm{Tr}\left[\gamma_\mu \frac{i}{\not{k} - \not{p} - m} \gamma_\rho \frac{i}{\not{k} - m} \gamma_\tau \frac{i}{\not{k} - \not{q} - m} \gamma_\nu \frac{i}{\not{k} - \not{q} - \not{p} - m} \right]$$

- Taking discontinuity of the box diagram contribution, we can get the absorptive part of scattering amplitude

$$(W_{\mu\nu\rho\tau})_{\mathrm{LO}} = \mathrm{Disc} A_{\mu\nu\rho\tau}$$

Calculation of One-loop diagram

- Applying the projection operator $(T_{TT})^{\mu\nu\rho\tau}$ $\times$



$$
\begin{aligned}
(T_{TT})^{\mu\nu\rho\tau} A_{\mu\nu\rho\tau} \quad = \quad & e^4(-)[c_{1b}B(1,-2,1,0) + c_{2b}B(1,-1,1,0) + c_{3b}B(1,1,1,0) \\
& + c_{4b}B(1,0,1,-2) + c_{5a}B(1,0,1,-1) + c_{6b}B(1,0,1,1) \\
& + c_{7b}B(1,0,1,0)] \cdots
\end{aligned}
$$

Non-positive → irrelevant

$$
B(\nu_1,\nu_2,\nu_3,\nu_4) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2-m^2]^{\nu_1}[(k-q)^2-m^2]^{\nu_2}[(k-p-q)^2-m^2]^{\nu_3}[(k-p)^2-m^2]^{\nu_4}}
$$

i.e. $c_{1b} = \dfrac{-8x^2(x^2+1)}{Q^4}$

- Taking the disconitinuities

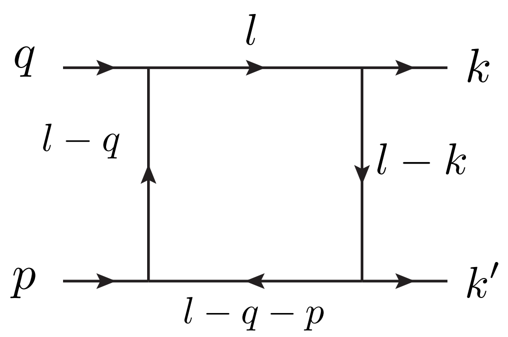
$$
\mathrm{Disc}B(0,\times,\times,\times) = \mathrm{Disc}B(\times,\times,0,\times) = 0
$$

$B(\nu_1,\nu_2,\nu_3,\nu_4)$, s are written in a linear combination of master integrals by reduction method

Reduction

- IBP identity $$\int d^D l \frac{\partial}{\partial l^\mu}\left(\frac{\eta^\mu}{D_1^{\nu_1} D_2^{\nu_2} D_3^{\nu_3} D_4^{\nu_4}}\right) = 0$$

$$\eta = l, l-q, l-p-q, l-k$$



From IBP identities

$$-(k-q)^2(\nu_2-1) = (D-\nu_{12344}+1)\mathbf{2}^- - (\nu_2-1)\mathbf{4}^- - (\nu_1\mathbf{1}^+ + \nu_3\mathbf{3}^+)\mathbf{2}^-\mathbf{4}^- \quad \text{etc.}$$

$\mathbf{1}^{+(-)}$:operator that increase(decrease) power of D_1

- Apply above operators to integral $\int d^D l \frac{1}{D_1^{\nu_1} D_2^{\nu_2} D_3^{\nu_3} D_4^{\nu_4}}$

- We get many relations among scalar integrals

  and solve these equations

S. Laporta('00)

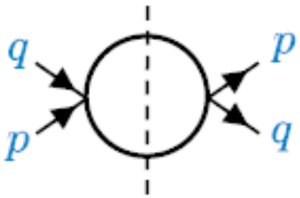
We can express a scalar integral as a linear combination of master integrals

Two-body phase-space integral

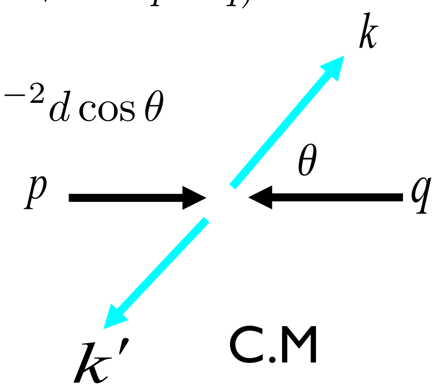
- Taking discontinuity of the master integrals

$$\int d\ PS^{(2)} = \int \frac{d^Dk}{(2\pi)^{D-1}}\delta^{(+)}(k^2-m^2)\int \frac{d^Dk'}{(2\pi)^{D-1}}\delta^{(+)}(k'^2-m^2)\delta^{(D)}(k+k'-p-q)$$

$$= \frac{1}{(16\pi)^{\frac{D}{2}-1}}\frac{s^{\frac{D}{2}-1}}{\Gamma(\frac{D}{2}-1)}\left(1-\frac{4m^2}{s}\right)^{\frac{D-3}{2}}\int_{-1}^{1}(1-\cos\theta)^{\frac{D}{2}-2}d\cos\theta$$



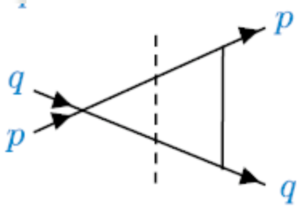
$$= \frac{\beta}{8\pi}$$



C.M frame

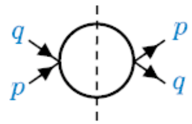
$$\beta = \sqrt{1-\frac{4m^2}{s}}$$

$$(k-q)^2 - m^2 = -p\cdot q(1+\beta\cos\theta)$$
$$(k-p)^2 - m^2 = -p\cdot q(1-\beta\cos\theta)$$



$$= \int dPS^{(2)}\frac{1}{(k-q)^2-m^2}$$

$$= \left(-\frac{Q^2}{2x}\right)\frac{1}{1+\beta}\ {}_2F_1\left(1-\epsilon,1,2-2\epsilon;\frac{2\beta}{1+\beta}\right) \times$$



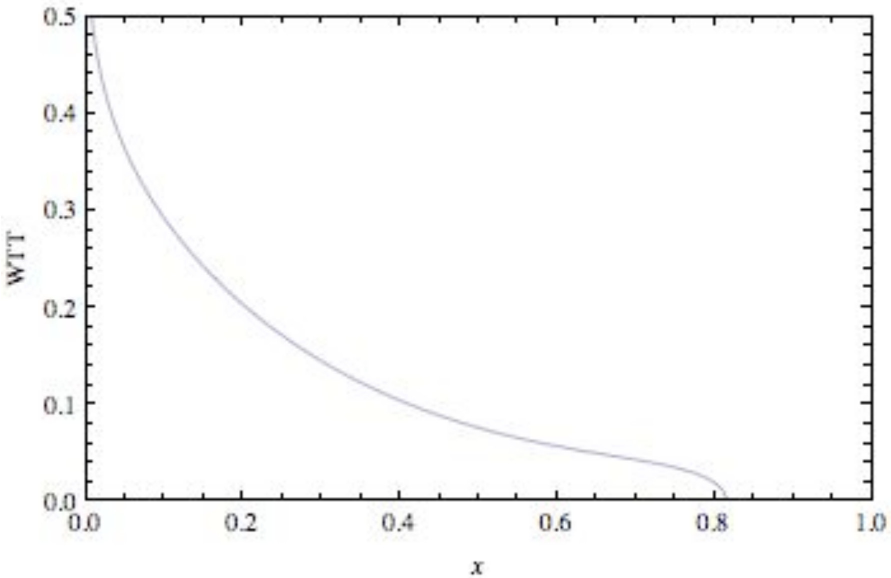
$$= -\frac{Q^2}{16x\pi}\log\left(\frac{1-\beta}{1+\beta}\right)$$

Hypergeometric fns.

Structure functions in leading order

$$(W_{TT})_{\rm LO} = \frac{\alpha}{2\pi}\delta_\gamma\Big[L\Big\{-8x^2\frac{m^4}{Q^4} + 4x(1-x)\frac{m^2}{Q^2} + 2x^2 + 2x + 1\Big\}$$
$$+\frac{\beta}{1-\beta^2}\Big\{-16x^2\frac{m^4}{Q^4} + 8x^2\frac{m^2}{Q^2} + (\beta^2-1)(1-x+x^2)\Big\}\Big]$$

Numerical plot



$$L = \log\frac{1-\beta}{1+\beta}$$

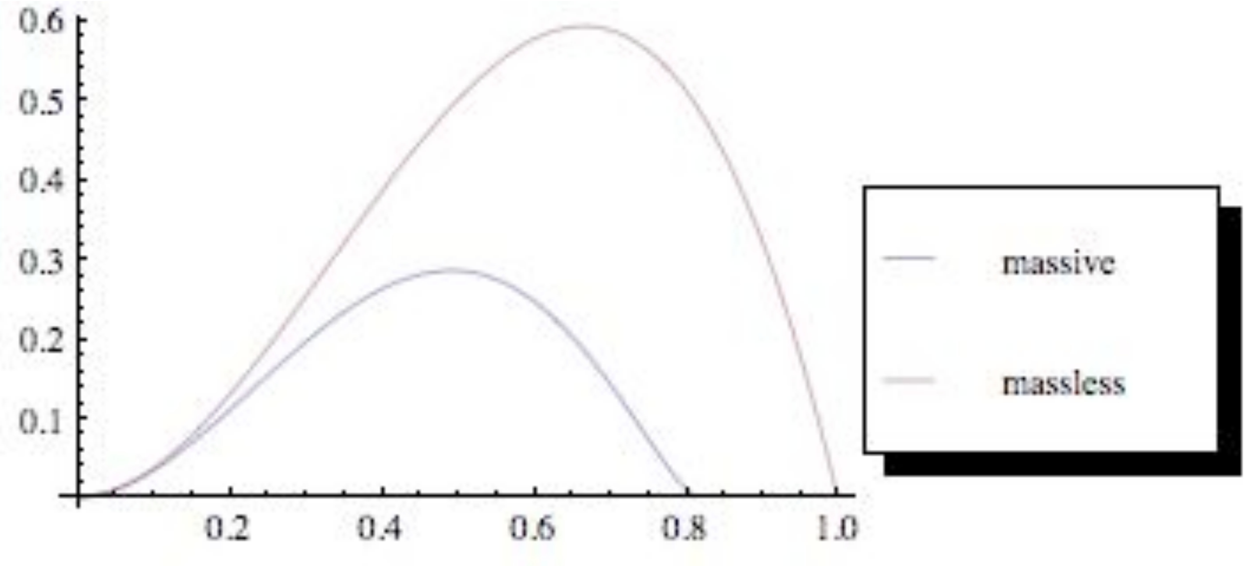
$$\delta_\gamma = 3\sum_{i=1}^{N_f} e_i^4$$

- Heavy quark mass threshold effect $x_{max} = \dfrac{1}{1+\frac{4m^2}{Q^2}}$ $(p+q)^2 \geq 4m^2$
- Heavy quark mass input $m_c = 1.3\text{GeV}$

Structure functions in leading order

$$F_L^\gamma = \frac{\alpha}{2\pi}\delta_\gamma \; x\Big[-8x^2 L\frac{m^2}{Q^2} + 4\beta x(1-x)\Big]$$

Numerical plot



- These results are consistent with the previous calculation

Calculation in NLO

Using dimensional regularization $(D = 4 - 2\epsilon)$

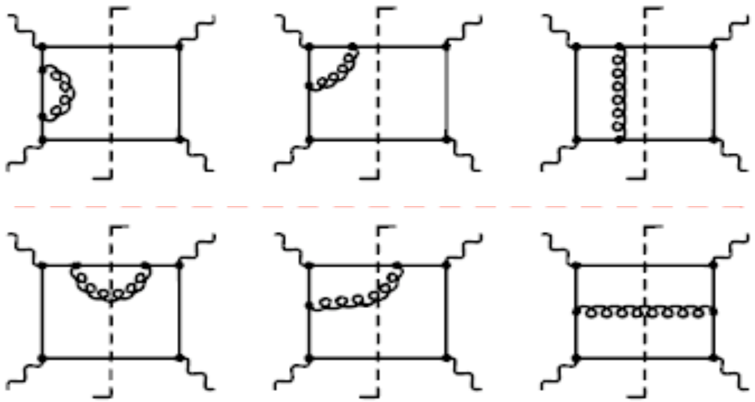
Infrared and Ultraviolet divergences appear

- Renormalization

- Kinoshita-Lee-Nauenburg theorem

Virtual correction

Real emission



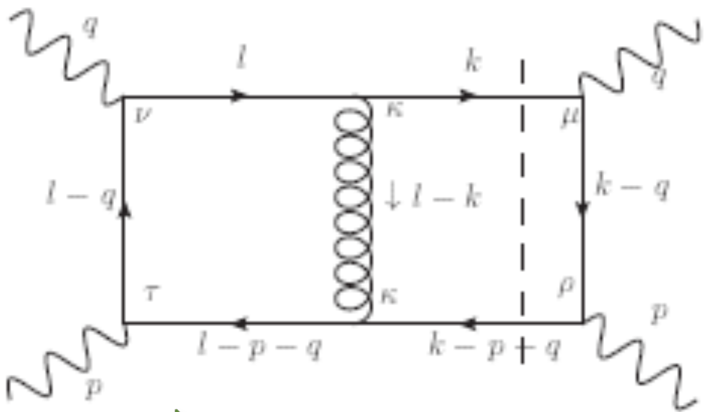
2-body phase space

3-body phase space

Virtual Corrections

- Virtual corrections have 2-body phase space integral
- UV are canceled by counter diagrams

How to calculate



- Using Cutkosky rule
- Taking Trace
- Applying Projection operators

⇒ Structure fns. can be expressed

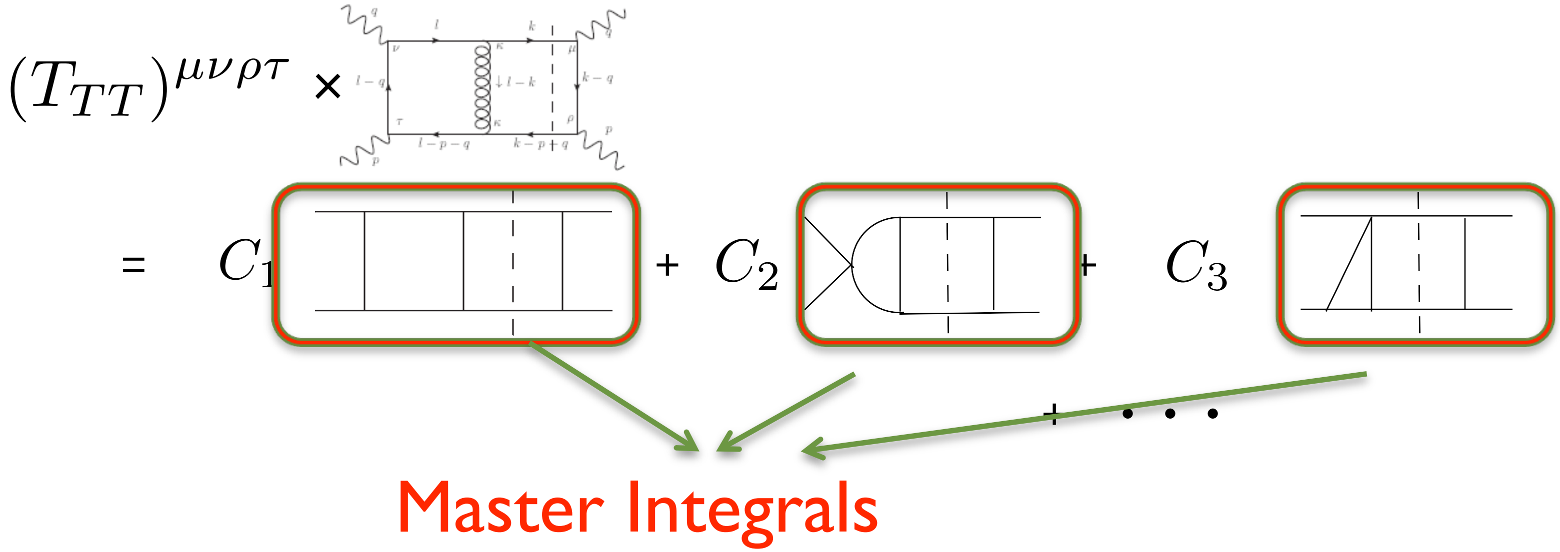
as a linear combination of scalar integrals

⇓

Select relevant scalar integrals only

- Using reduction method, the relevant scalar integrals reduce to the smaller number of master integrals

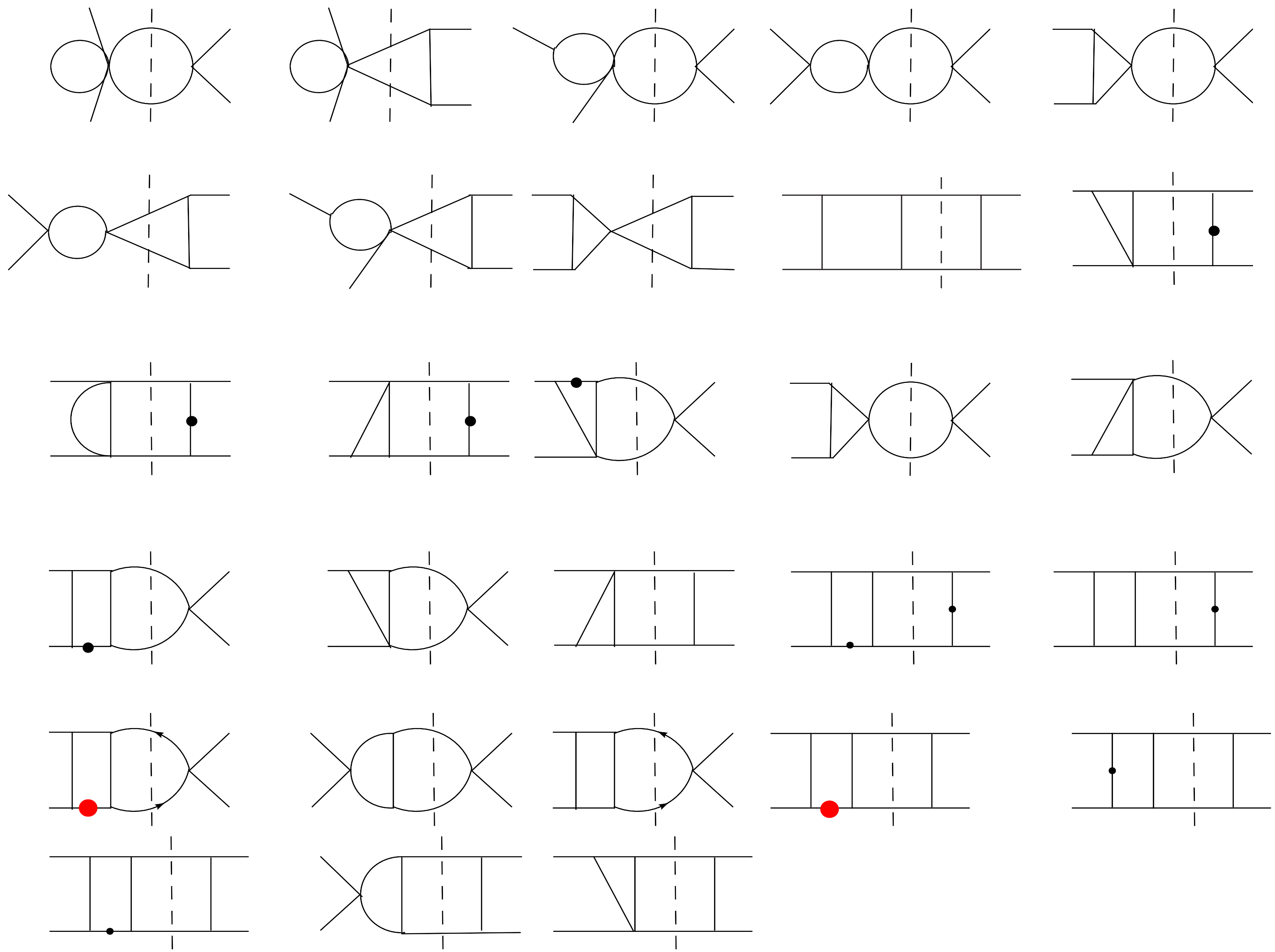
- We perform reduction of two-loop integrals



- Structure function can be written in a linear combination of master integrals
  - ◆ For this typical graph, UV divergence does not appear, but IR divergence appears

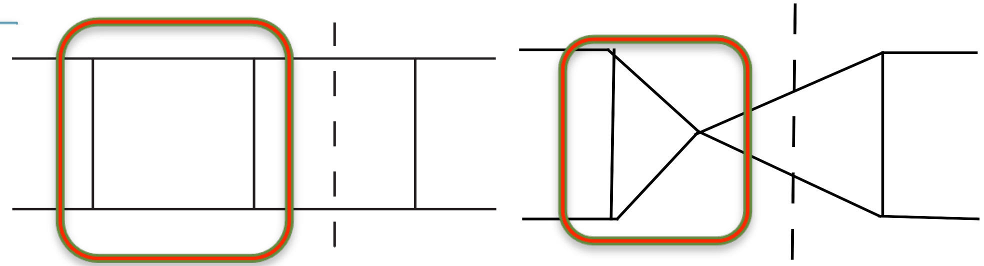
- As a check, the sum of UV from master integrals should vanish

Master Integrals



- Virtual corrections ------ 23 master integrals

Analytic continuation



- When the one-loop integrals $\int d^D l \frac{1}{[l^2 - D]^n}$

We assume $s, \; q^2$ are space-like

$$s < 0, \;\; q^2 < 0$$

D is positive definite

The $i\epsilon$ prescription can be dropped

- The analytic continuation is performed by restoring the $i\epsilon$

$$s \longrightarrow s + i\epsilon$$

- The one-loop integrals are expressed in terms of two types of transcendental functions

$$\ln\Big(\prod_{i=1}^{n} x_i\Big), \quad \mathrm{Li}_2\Big(1 - \prod_{i=1}^{n} x_i\Big)$$

Analytic continuation for logarithm

$$\ln\Big(\prod_{i=1}^{n} x_i\Big) \to \sum_{i=1}^{n} \ln(x_i)$$

Analytic continuation for dilogarithm

For $|\prod_{i=1}^{n} x_i| < 1$

$$\mathrm{Li}_2\Big(1-\prod_{i=1}^{n} x_i\Big) \longrightarrow \mathrm{Li}_2\Big(1-\prod_{i=1}^{n} x_i\Big) + \ln\Big(1-\prod_{i=1}^{n} x_i\Big)\Big[\ln\Big(\prod_{i=1}^{n} x_i\Big) - \sum_{i=1}^{n} \ln(x_i)\Big]$$

$$= \frac{\pi^2}{6} - \mathrm{Li}_2\Big(\prod_{i=1}^{n} x_i\Big) - \ln\Big(1-\prod_{i=1}^{n} x_i\Big)\sum_{i=1}^{n} \ln(x_i)$$

For $|\prod_{i=1}^{n} x_i| > 1$

$$\mathrm{Li}_2\Big(1-\prod_{i=1}^{n} x_i\Big) = -\mathrm{Li}_2\Big(1-\frac{1}{\prod_{i=1}^{n} x_i}\Big) - \frac{1}{2}\ln^2\Big(\prod_{i=1}^{n} x_i\Big)$$

Master Integral

- UV divergence comes from gamma functions

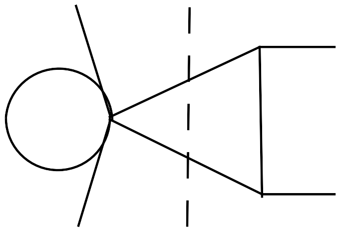
$$\int d^D k \quad \sim \quad \Gamma(\epsilon)$$

- IR divergence comes from Feynman parameter integrals

$$\int_0^1 da \frac{1}{(1-a)^{1+\epsilon}}$$

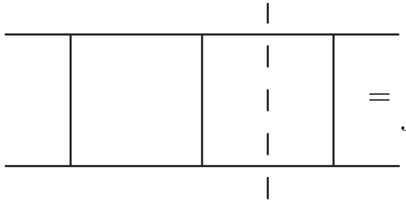
- Most of the absorptive part of master integrals can be integrated analytically

$$\alpha = \sqrt{1+\frac{4m^2}{Q^2}} \qquad \beta = \sqrt{1-\frac{4m^2}{s}}$$



$$= \int dPS^{(2)} \frac{1}{(k-q)^2 - m^2 + i\epsilon} \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - m^2 + i\epsilon}$$

$$= iS^{2\epsilon}\left(\frac{1}{m^2(s-4m^2)}\right)^{\epsilon} \frac{1}{128pi^3} \frac{-xm^2}{Q^2} \left\{\frac{1}{\epsilon}+1\right\}\left\{\log\left(\frac{1+\beta}{1-\beta}\right) + \epsilon\left[\frac{1}{2}\log^2\frac{1+\beta}{1-\beta} + 2\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right)\right]\right\}$$



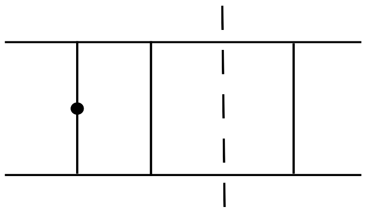
$$= \int dPS^{(2)} \frac{1}{(k-q)^2 - m^2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2 - m^2 + i\epsilon][(l-q)^2 - m^2 + i\epsilon][(l-p-q)^2 - m^2 + i\epsilon](l-k)^2}$$

IR div

$$= iS^{2\epsilon}\left(\frac{1}{m^2(s-4m^2)}\right)^{\epsilon} \frac{1}{128\pi^3} \frac{-xs}{2Q^2} B(1-\epsilon, 1-\epsilon) \ln\left(\frac{1-\beta}{1+\beta}\right)\Bigg\{\boxed{\frac{1}{\epsilon}} \ln\left(\frac{1-\beta}{1+\beta}\right) - 2\ln\left(\frac{1-\beta}{1+\beta}\right)\left[1 - \ln\left(\frac{m^2}{p\cdot q}\right)\right]$$

$$-\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \ln^2\left(\frac{\alpha-1}{\alpha+1}\right) - \mathrm{Li}_2\left(\frac{2\beta}{(1+\beta)^2}\right)$$

$$+2\mathrm{Li}_2\left(\frac{2(\alpha+\beta)}{(1+\beta)(\alpha+1)}\right) + 2\mathrm{Li}_2\left(\frac{2(\alpha-\beta)}{(1+\beta)(\alpha-1)}\right) + (\log^2(1+\beta) - \log^2(1-\beta))\Bigg\}$$



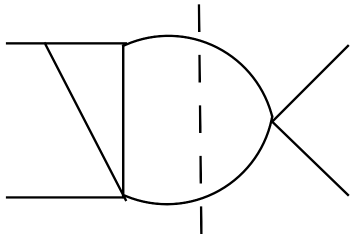
- Integral with UV and IR divergence

$$= iS^{2\epsilon}\left(\frac{1}{m^2(s-4m^2)}\right)^{\epsilon} \frac{\beta}{128\pi^3}\Bigg\{\left(\frac{-2x}{Q^2}\right)\frac{1}{2\beta}\left\{\log\frac{1+\beta}{1-\beta} + \epsilon\left[\frac{1}{2}\log^2\frac{1+\beta}{1-\beta} + 2\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right)\right]\right\}$$

$$\times\left\{\left\{\frac{1}{\epsilon_U} + \left[3 + \sqrt{1-\frac{4m^2}{s}}\ \log x_s\right] - \frac{1}{2}\right\} - \left\{\left(3 - \frac{q^2}{m^2}\right)\log x_s - \frac{1}{2}\left(1-\frac{1}{x}\right)\frac{q^2}{m^2}\left[\sqrt{1-\frac{4m^2}{s}} - \frac{2m^2}{s}\log x_s\right]\right\}\right\}$$

$$-(1+2\epsilon)\frac{1}{m^2}\left(\frac{Q^2}{-2x}\right)\left[\frac{\log x_s}{\epsilon_I} + \log x_s\left(2 + \frac{1}{2}\log x_s - 2\log(1-x_s^2)\right) - \mathrm{Li}_2(x_2^2) - 2\mathrm{Li}_2(1-x_s) + \frac{\pi^2}{6}\right]\Bigg\}$$

- For some of the mater integrals, it is difficult to perform the phase-space integral analytically,
  We perform numerical integration for these integrals



$$= \int dPS^{(2)} \frac{1}{[l^2 - m^2 + i\epsilon][(l-q)^2 - m^2 + i\epsilon](l-k)^2}$$

$$= \int dPS^{(2)}(-iS^{\epsilon})\frac{1}{16pi^2}\left(\frac{1}{m^2}\right)^{\epsilon}\Bigg[\frac{1}{m^2}\frac{Q^2}{\gamma_1-\gamma_2}\Big\{\mathrm{Li}_2\,(1) - \mathrm{Li}_2\left(\frac{\gamma_1}{\gamma_1-\lambda_2}\right) - \mathrm{Li}_2\left(-\frac{\gamma_1}{\lambda_1-\gamma_1}\right) - \mathrm{Li}_2\left(-\frac{1-\gamma_1}{\gamma_1}\right)$$
$$+\mathrm{Li}_2\left(-\frac{1-\gamma_1}{\gamma_1-\lambda_2}\right) + \mathrm{Li}_2\left(\frac{1-\gamma_1}{\lambda_1-\gamma_1}\right) - \log\left(\frac{m^2-t}{m^2}\right)\log\left(1-\frac{1}{\gamma_2}\right) + \log\left(\frac{\alpha+1}{\alpha-1}\right)\log\left(\frac{\lambda_1-\gamma_2}{\lambda_2-\gamma_2}\right) - \mathrm{Li}_2\left(\frac{1}{\gamma_2}\right)$$
$$-\mathrm{Li}_2\left(\frac{\lambda_2}{\lambda_2-\gamma_2}\right) + \mathrm{Li}_2\left(\frac{\lambda_2-1}{\lambda_2-\gamma_2}\right) + \mathrm{Li}_2\left(\frac{\lambda_1-1}{\lambda_1-\gamma_2}\right) - \mathrm{Li}_2\left(\frac{\lambda_1}{\lambda_1-\gamma_2}\right)\Big\}\Bigg]$$

$$\lambda_1,\ \lambda_2 = \frac{1 \pm \alpha}{2} = \mathrm{const}$$

$$\gamma_1 = \frac{Q^2 - m^2 + (k-q)^2 + \sqrt{(m^2 - (k-q)^2 - Q^2)^2 - 4Q^2m^2}}{2Q^2}$$

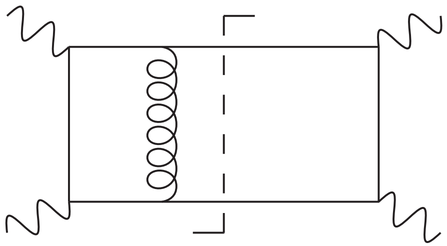
Contribute to phase-space integral

$$\gamma_2 = \frac{Q^2 - m^2 + (k-q)^2 - \sqrt{(m^2 - (k-q)^2 - Q^2)^2 - 4Q^2m^2}}{2Q^2}$$

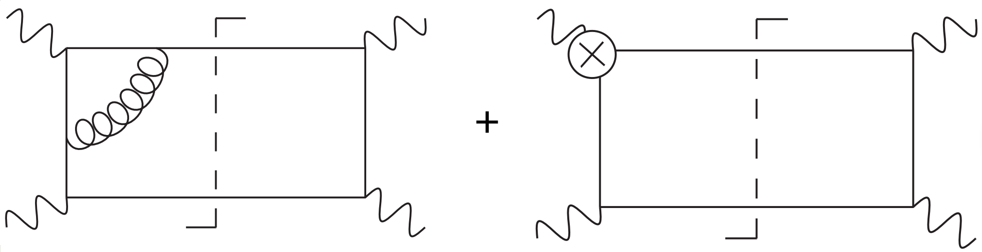
$$\boxed{\begin{aligned}(k-q)^2 - m^2 &= -p\cdot q(1+\beta\cos\theta)\\ (k-p)^2 - m^2 &= -p\cdot q(1-\beta\cos\theta)\end{aligned}}$$

Summary of virtual gluon corrections

- Using cutkosky rule and reduction method,
  structure fns. are written in terms of master integrals.
- Master integrals are evaluated analytically and numerically



- UV are canceled
- IR are remained

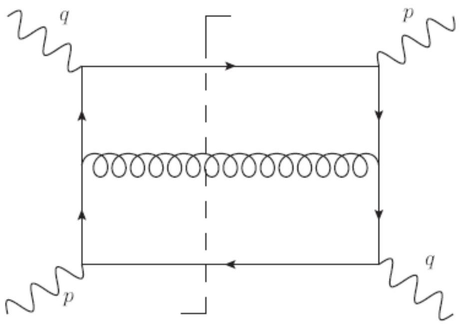


- UV are canceled
- IR do not appear

....

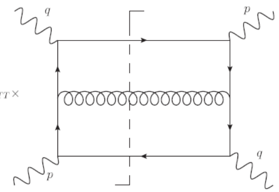
Real gluon emission

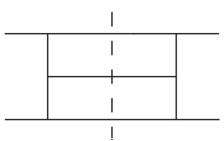
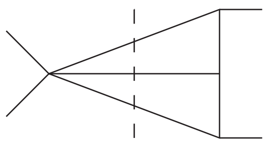
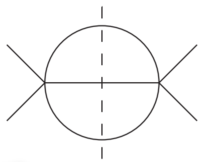
- Real emission diagrams have 3-body phase space integrals



- The procedure of the calculation is same as the case of the virtual corrections

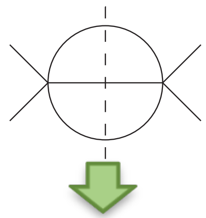
  Cutkosky rule, projection operator, reduction

$(T_{TT})^{\mu\nu\rho\tau}$ $\times$ 

$=$ A_1  $+$ A_2  $+ A_3$  $\cdots$

Master Integrals

Three-body phase-space integral



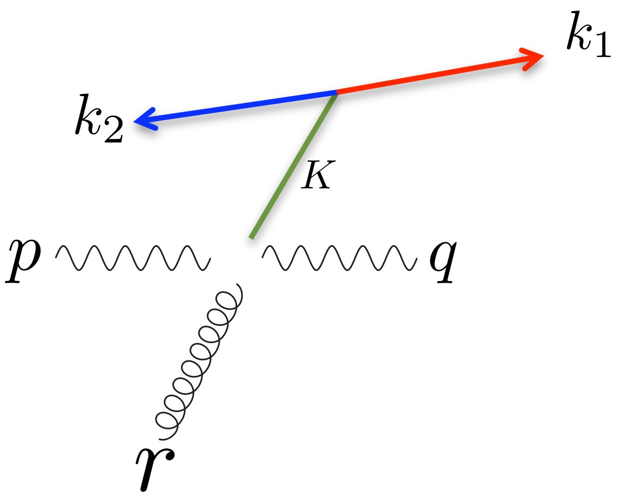
$$\int dPS^{(3)} = \int \frac{d^D r}{(2\pi)^{D-1}} \frac{d^D k_1}{(2\pi)^{D-1}} \frac{d^D k_2}{(2\pi)^{D-1}} (2\pi)^D \delta^D(q+p-r-k_1-k_2)\delta^+(r^2)\delta^+(k_1^2-m^2)\delta^+(k_2^2-m^2)$$

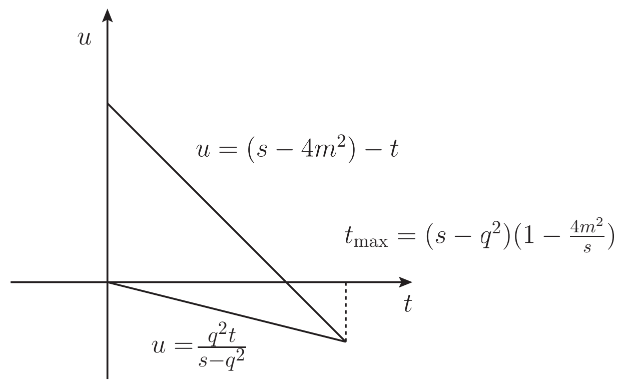
$$= \frac{1}{(4\pi)^D}\frac{1}{\Gamma(D-3)}(s-q^2)^{3-D}\int_0^\pi d\theta \sin^{D-3}\theta \int_0^\pi d\phi \sin^{D-4}\phi \int_0^{(s-q^2)(1-\frac{4m^2}{s})} dt \int_{q^2t/(s-q^2)}^{(s-4m^2)-t} du$$

$$\left(1-\frac{4m^2}{s-t-u}\right)^{\frac{D-3}{2}} \left\{(s-t-u)\ t\ \Big[(s-q^2)u-q^2t\Big]\right\}^{\frac{D-4}{2}}$$

$$= \frac{1}{(4\pi)^D}\frac{s^{D-3}}{\Gamma(D-3)}\left(1-\frac{4m^2}{s}\right)^{\frac{3D-7}{2}} \int_0^\pi d\theta \sin^{D-3}\theta \int_0^\pi d\phi \sin^{D-4}\phi$$

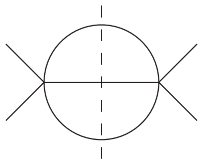
$$\times \int_0^1 dy \int_0^1 dz \left\{\frac{4m^2}{s} + (1-\frac{4m^2}{s})(1-y)z\right\}^{-\frac{1}{2}} y^{\frac{D-4}{2}}(1-y)^{D-\frac{5}{2}}\ z^{\frac{D-3}{2}}(1-z)^{\frac{D-4}{2}}$$





Master integral for 3-body phase-space

- The easiest master integral



$$= \int dPS^{(3)}$$

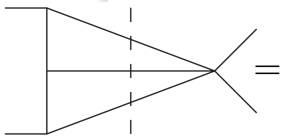
$$= S^{2\epsilon}\,\frac{s}{128\pi^3}\left(1-\frac{4m^2}{s}\right)^{\frac{5}{2}}\left\{1-\epsilon\left[3\log\left(1-\frac{4m^2}{s}\right)+2\log s-\psi^{(0)}\left(\frac{3}{2}\right)+\psi^{(0)}\left(\frac{1}{2}\right)\right]+\mathcal{O}(\epsilon^2)\right\}\ \ S_1\left(\frac{s}{4m^1}\right)$$

$$S_1(a)=\frac{1}{4(1-a)^{\frac{5}{2}}}\Big\{-3a^2\log[a(1-a)]+2a^2\log[a+\sqrt{1-a}-1]-a^2(\tanh^{-1}\sqrt{1-a})$$
$$+a\sqrt{1-a}+2\sqrt{1-a}-4a(1-a)\log[-a+\sqrt{1-a}+1]+2a\log[a(1-a)]\Big\}$$

can be evaluated analytically

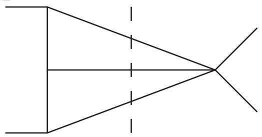
$$\beta_{12}=\beta_{12}(t,u)$$

- Other master integrals are difficult to calculate analytically



$$=\frac{1}{(4\pi)^D}\frac{1}{\Gamma(D-3)}(s-q^2)^{3-D}\int_0^{\pi}d\theta\sin^{D-3}\theta\int_0^{\pi}d\phi\sin^{D-4}\phi\int_0^{(s-q^2)(1-\frac{4m^2}{s})}dt\int_{q^2t/(s-q^2)}^{(s-4m^2)-t}du$$

$$\left(1-\frac{4m^2}{s-t-u}\right)^{\frac{D-3}{2}}\left\{(s-t-u)\ t\ \left[(s-q^2)u-q^2t\right]\right\}^{\frac{D-4}{2}}\frac{-2}{s-t-q^2}\,\frac{1}{\{1+\beta_{12}\cos\theta\}}$$



$$= \frac{s}{128\pi^3}\Big(1-\frac{4m^2}{s}\Big)^{\frac{5}{2}} \times \frac{-1}{s\Big(1-\frac{4m^2}{s}\Big)^{\frac{1}{2}}}(1-x)\times S_{D0d}\Big(\frac{4m^2}{s},x\Big)$$

$$S_{D0d}(a,x) \equiv \int_0^1 dy \int_0^1 dz \frac{1-y}{\sqrt{\widetilde{A}}}\log\left|\frac{2A+B+2\sqrt{A(A+B+C)}}{-2A+B+2\sqrt{A(A-B+C)}}\right|$$

$$\widetilde{A} \;=\; \Big[1-y(1-a)\Big]^2 + 2(1-x)(1-a)\Big[-(1-a)y^2z + y(1+a-az)+z-1\Big] + (1-x)^2(1-a)^2\Big[1-(1-y$$

$$A = s^2\frac{1-a}{(1-x)^2}\;\frac{(1-y)z}{a+(1-a)(1-y)z}\Big\{\Big[1-y(1-a)\Big]^2 + 2(1-x)(1-a)\Big[-(1-a)y^2z+y(1+a-az)+z-1\Big]+(1-x)^2(1-a)^2\Big[1-(1-y)z\Big]^2\Big\}$$

$$B = 2s^2\sqrt{\frac{(1-a)(1-y)z}{a+(1-a)(1-y)z}}\frac{(1-a)(1-x)(1-y)z+(1-a)(x+y)+a}{(1-x)^2[1-y(1-a)]}\Big\{\Big[1-y(1-a)\Big]^2-(1-x)(1-a)\Big[(1-y)(1-z)-y\Big\}$$

$$\sqrt{A+B+C} = \frac{s}{(1-x)}\Big[(1-a)(1-x)(1-y)z+(1-a)(x+y)+a\Big] + \frac{s}{(1-x)[1-y(1-a)]}\quad\sqrt{\frac{(1-a)(1-y)z}{a+(1-a)(1-y)z}}$$
$$\times\Big\{\Big[1-y(1-a)\Big]^2-(1-x)(1-a)\Big[(1-y)(1-z)-y\{a+(1-a)(1-y)z\}\Big]\Big\}$$

- Most of the master integrals for real emission are hard to be calculated analytically

- Numerical analysis of these master integrals are in progress.

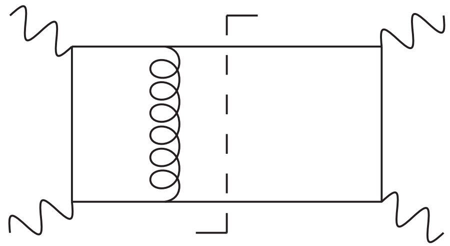
Summary

- The heavy quark mass effects on the photon structure functions are investigated in parton model up to the NLO
- Using Cutkosky rule and reduction method, structure functions are expressed in terms of the master integrals.
- The master integrals for virtual corrections are evaluated

Ultraviolet divergence are canceled

Infrared divergence are remained

should be canceled when contribution of real gluon emission diagrams is added



- Analysis of the real gluon emission diagrams in progress